# 2 Could Minkowski have discovered the cause of gravitation before Einstein?

Vesselin Petkov

**Abstract** There are two reasons for asking such an apparently unanswerable question. First, Max Born's recollections of what Minkowski had told him about his research on the physical meaning of the Lorentz transformations and the fact that Minkowski had created the full-blown four-dimensional mathematical formalism of spacetime physics before the end of 1907 (which could have been highly improbable if Minkowski had not been developing his own ideas), both indicate that Minkowski might have arrived at the notion of spacetime independently of Poincaré (who saw it as nothing more than a mathematical space) and at a deeper understanding of the basic ideas of special relativity (which Einstein merely postulated) independently of Einstein. So, had he lived longer, Minkowski might have employed successfully his program of regarding four-dimensional physics as spacetime geometry to gravitation as well. Moreover, Hilbert (Minkowski's closest colleague and friend) had derived the equations of general relativity simultaneously with Einstein. Second, even if Einstein had arrived at what is today called Einstein's general relativity before Minkowski, Minkowski would have certainly reformulated it in terms of his program of geometrizing physics and might have represented gravitation *fully* as the manifestation of the non-Euclidean geometry of spacetime (Einstein regarded the geometrical representation of gravitation as pure mathematics) exactly like he reformulated Einstein's special relativity in terms of spacetime.

## 1 Introduction

On January 12, 1909, only several months after his Cologne lecture *Space and Time* [1], at the age of 44 Hermann Minkowski untimely left this world. We will never know how physics would have developed had he lived longer. What seems undeniable is that the discovery of the link between gravitation and the non-Euclidean geometry of spacetime might have been quite different

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from what had actually happened.

On the one hand, Einstein's way of thinking based on conceptual analyses and thought experiments now seems to be perhaps the only way powerful enough to decode the unimaginable nature of gravitation. Indeed in 1907 (most probably in November) Einstein had already been well ahead of Minkowski in terms of deeply thinking of the apparently self-evident manifestations of gravitational phenomena when he made a gigantic step towards the new theory of gravitation [2]:

> I was sitting in a chair in the patent office at Bern when all of a sudden a thought occurred to me: "If a person falls freely he will not feel his own weight." I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation.

Einstein had been so impressed by this insight that he called it the "happiest thought" of his life [2]. And indeed this is a crucial point – at that time it seemed Einstein had been the only human who realized that no gravitational force acted on a falling body (in fact, as we will see is Section 3 Einstein might have misinterpreted his happiest thought). Then he struggled for eight years to come up with a theory – his general relativity – according to which (as we see it today) gravity is not a force but a manifestation of the curvature of spacetime.

On the other hand, however, Minkowski's three papers on relativity, particularly his Cologne lecture *Space and Time* revealed that in the reformulation of Einstein's special relativity he employed a powerful research strategy (rivaling Einstein's research strategy) – exploring the internal logic of the mathematical formalism of physical theories. That is why, had he lived longer, Minkowski and his closest colleague and friend David Hilbert might have formed an unbeatable team in theoretical physics and might have discovered general relativity (surely under another name) before Einstein. Moreover, contrary to common belief, as Lehmkuhl showed [3], Einstein himself did not believe that general relativity geometrized gravitation: "I do not agree with the idea that the general theory of relativity is geometrizing Physics or the gravitational field" [4].

As there is no way to reconstruct what might have happened in the period 1909-1915 I will outline here what steps had been logically available to Minkowski on the basis of his results. I will imagine two logically possible scenarios. In Section 2 I will describe how Minkowski, while employing his program of geometrizing physics to gravitation, might have realised that gravitational phenomena may be manifestations of a non-Euclidean geometry of spacetime. In Section 3 I will discuss the possibility that it was Einstein who first realized that gravitation can be described in terms of non-Euclidean geometry, but since he regarded the geometrization of gravitation only as a mathematical tool, Minkowski might have reformulated Einstein's general relativity by demonstrating that gravitation is not a physical interaction but just curved-spacetime geometry.

## 2 First scenario

In order to understand better what Minkowski could have done, had he lived longer, it is important to take explicitly into account two indications of why he appears to have realized independently the equivalence of the times of inertial observers in relative motion (what Einstein postulated and which formed the basis of his special relativity) and that the Lorentz transformations can be regarded as rotations in a four-dimensional world (which was first published by Poincaré but he did not see anything revolutionary in that observation since he believed that physical theories do not necessarily represent anything in the physical world since they are nothing more than *convinient descriptions* of physical phenomena).

These two intications are:

- Max Born's recollections of what Minkowski had told him about his research on the physical meaning of the Lorentz transformations and about his shock when Einstein published his 1905 paper in which he postulated the equivalence of different local times of observers in relative motion.
- What is far more important than Born's recollections is the fully-developed four-dimensional formalism describing an absolute four-dimensional world, which Minkowski reported on December 21, 1907 and the depth of his understanding of the electrodynamics of moving bodies. Such a revolution in both physics and mathematics could not have been possible if Minkowski had not been developing his own ideas but had to first understand Einstein's 1905 paper even better than Einstein in order to invent that formalism to reformulate his theory as a theory of an absolute four-dimensional world. Born's recollections simply confirm what appears to be the most probable history of spacetime physics – that Minkowski independently discovered (i) the equivalence of the times of inertial observers in relative motion, and (ii) the notion of spacetime, but Einstein and Poincaré published their results first.

Here is the historical context of Minkowski's comments reflected in Born's recollections.

By 1905 Minkowski was already internationally recognized as an exceptional mathematical talent – in 1883 he received (with Henry Smith) the French Academy's Grand Prize in Mathematics for his innovative geometric approach to the theory of quadratic forms and in 1896 he published his major work in mathematics *The Geometry of Numbers* [5].

At that time Minkowski was already interested in an unresolved issue at the very core of fundamental physics – at the turn of the nineteenth and twentieth century Maxwell's electrodynamics showed that light is an electromagnetic wave, which seemed to imply that it propagates in a light carrying medium (the luminiferous ether), but its existence had been put into question since the Michelson and Morley interference experiment failed to detect the Earth's motion in that medium. This puzzling result was in full agreement with the experimental impossibility to detect absolute uniform motion with mechanical means captured in Galileo's principle of relativity – absolute uniform motion cannot be detected by mechanical experiments. The Michelson and Morley experiment showed that absolute uniform motion cannot be detected by elecromagnetic experiments either.

Minkowski's documented active involvement with the electrodynamics of moving bodies began in the summer of 1905 when he and his friend David Hilbert co-directed a seminar in Göttingen on the electron theory (dealing with the electrodynamics of moving bodies). Einstein's paper on special relativity was not published at that time; *Annalen der Physik* received the paper on June 30, 1905. Poincaré's longer paper *Sur la dynamique de l'électron* was not published either; it appeared in 1906. Also, "Lorentz's 1904 paper (with a form of the transformations now bearing his name) was not on the syllabus" [6].

Minkowski's student Max Born, who attended the seminar in 1905, wrote [7]:

> We studied papers by Hertz, Fitzcerald, Larmor, Lorentz, Poincaré, and others but also got an inkling of Minkowski's own ideas which were published only two years later.

Born also recalled what specifically Minkowski had said during the seminar (quoted in [8]):

> I remember that Minkowski occasionally alluded to the fact that he was engaged with the Lorentz transformations, and that he was on the track of new interrelationships.

Again Born wrote in his autobiography about what Minkowski had told him after Minkowski's lecture *Space and Time* given on September 21, 1908 [9]:

> He told me later that it came to him as a great shock when Einstein published his paper in which the equivalence of the different local times of observers moving relative to each other were pronounced; for he had reached the same conclusions independently but did not publish them because he wished first to work out the mathematical structure in all its splendour. He never made a priority claim and always gave Einstein his full share in the great discovery.

An additional indication that Minkowski did not just reformulate Einstein's special relativity in terms of spacetime, but that he discovered the spacetime physics[1] by independently realizing (i) the equivalence of the times

[1] That Minkowski had indeed been developing his own ideas and independently formulated the physics of spacetime is confirmed by Born's recollections above – the first two show that Minkowski was already discussing his own ideas at the seminar in the summer of 1905. Note that at that time Einstein's 1905 paper was not published; Minkowski asked Einstein to send him the 1905 paper hardly on October 9, 1907 [10]. It appears Minkowski needed two years – from 1905 to 1907 – to develop the full mathematical formalism of the four-dimensional physics of spacetime introduced by him (published in 1908 as a 59-page treatise [11]).

of inertial observers in relative motion, and (ii) the meaning of the Lorentz transformations (by successfully decoding the profound physical message hidden in the failed experiments to detect absolute uniform motion) is the fact that Minkowski *explained* what Einstein merely *postulated.* Einstein postulated:

– The equivalence of the time of a "stationary" observer and the different time of a moving observer (formally introduced as an auxiliary mathematical notion by Lorentz).

– The experimental impossibility to detect absolute motion (captured in the relativity postulate).

– That the speed of light is the same in all inertial reference frames.

Minkowski explained (see Minkowski's paper [1] and also [13] and in this section):

– The equivalence of the times of inertial observers in relative motion – *why* such observers have different times.

– The relativity postulate – *why* absolute motion (with constant velocity) cannot be detected or its modern formulation – *why* the laws of physics are the same in all inertial reference frames.

– *Why* the speed of light is the same in all inertial reference frames.

It seems it took some time for Einstein to unterstand Minkowski's spacetime physics as implied by Sommerfeld's recollection of what Einstein said on one occasion which reveals Einstein's initial rather hostile attitude towards Minkowski's work: "Since the mathematicians have invaded the relativity theory, I do not understand it myself any more" [14]. Despite his initial negative reaction towards Minkowski's four-dimensional physics Einstein relatively quickly realized that his revolutionary theory of gravitation would be impossible without the revolutionary spacetime physics discovered by Minkowski. At the beginning of his 1916 paper on general relativity Einstein wrote: "The generalization of the theory of relativity has been facilitated considerably by Minkowski, a mathematician who was the first one to recognize the formal equivalence of space coordinates and the time coordinate, and utilized this in the construction of the theory"[2] [15].

To understand fully what logical options would have been realistically available to Minkowski in 1909, one has to realize that Minkowski regarded the unification of space and time into *die Welt* – a four-dimensional world – as real. This is important not only to understand what Minkowski could have done had he lived longer, but because the issue of the reality of spacetime (Minkowski's four-dimensional world) constitutes an unprecedented situation in fundamental physics. It seems many physicists, including relativists, simply refuse to see the double experimental proof of the reality of spacetime. The first experimental proof is the set of all experiments (including the Michelson and Morley experiment) that failed to detect absolute uniform

[2] This quote is hardly from the new 1997 translation [15]. Quite strangely, the first page of the paper containing the recognition of Minkowski's work had been omitted in the first English translation [18].

motion and that gave rise to the relativity postulate. It is these experiments whose hidden profound message was successfully decoded by Minkowski – absolute (uniform) motion cannot be detected because such a thing does not exist in Nature; absolute motion presupposes absolute (i.e. single) space, but those experiments imply that observers in relative motion have different times and spaces, which in turn implies that the world is a four-dimensional world.

On September 21, 1908 Minkowski explained how he decoded the profound message hidden in the failed experiments to discover absolute motion in his famous lecture *Space and Time* and announced the revolutionary view of space and time, which he deduced from those experiments [1, p.111]:

> The views of space and time which I want to present to you arose from the domain of experimental physics, and therein lies their strength. Their tendency is radical. From now onwards space by itself and time by itself will recede completely to become mere shadows and only a type of union of the two will still stand independently on its own.

Here is Minkowski's most general proof that the world is four-dimensional. To explain the experiment of Michelson and Morley, which failed to detect the Earth's absolute motion, Lorentz suggested that observers on Earth can formally use a time that is different from the true time of an observer at absolute rest. Einstein postulated that the times of different inertial observers in relative motion are equally good, that is, each observer has his own time, and that for Einstein meant that time is relative.

Minkowski demonstrated that as observers in relative motion have different equally real times, they inescapably have *different spaces* as well, because space is defined as a set of simultaneous events, and different times imply different sets of simultaneous events, i.e., different spaces[3] (or simply – different times imply different spaces because space is perpendicular to time) [1, p. 114]:

> "Hereafter we would then have in the world no more *the* space, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter in four-dimensional physics. You see why I said at the beginning that space and time will recede completely to become mere shadows and only a world in itself will exist."

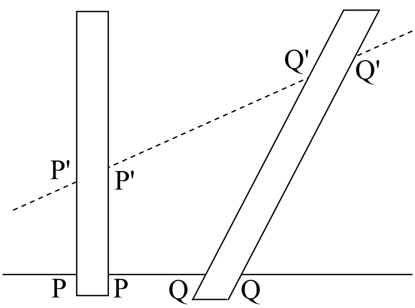
Therefore the experimental failure to detect absolute motion has indeed a profound physical meaning – that there exists not a single (and therefore absolute) space, but many spaces (and many times). As many spaces are

[3]Minkowski specifically tried to explain why "the concept of space was shaken neither by Einstein nor by Lorentz" [1, p. 117] which prevented them from discovering the truly revolutionary spacetime physics.

possible in a four-dimensional world, Minkowski's irrefutable proof that the world is four-dimensional becomes self-evident:[4]

*If the real world were three-dimensional, there would exist a* ***single*** *space, i.e. a single class of simultaneous events (and therefore a single time), which would mean that simultaneity and time would be absolute in contradiction with both the theory of relativity and, most importantly, with the experiments which failed to detect absolute motion.*

The second experimental proof of the reality of spacetime are all experiments that confirmed the kinematic relativistic effects. How these experiments would be *impossible* if the world were *not* four-dimensional (i.e., if spacetime were just a mathematical space) is immediately seen in Minkowski's own explanation of length contraction (which is the accepted explanation) – as length contraction (along with time dilation) is a specific manifestation of relativity of simultaneity, an assumption that reality is *not* a four-dimensional world directly leads (as in the above paragraph) to absolute simultaneity (and to the impossibility of length contraction [16]) in contradiction with relativity and the experiments that confirmed length contraction; one of the experimental tests of length contraction (along with time dilation) is the muon experiment – "in the muon's reference frame, we reconcile the theoretical and experimental results by use of the length contraction effect, and the experiment serves as a verification of this effect" [17].




The right half of Figure 1 of Minkowski's paper *Space and Time*

To see exactly how length contraction would be impossible if reality were a three-dimensional world, consider Minkowski's explanation whose essence is that length contraction of a body is a manifestation of the reality of the body's worldtube. Minkowski considered two bodies in uniform relative motion represented by their worldtubes in the figure above (see Figure 1 of Minkowski's paper [1]). Consider only the body represented by the vertical worldtube to understand why the worldtube of a body must be real in order that length contraction be possible. The three-dimensional cross-section $PP$, resulting from the intersection of the body's worldtube and the space (repre-

[4]Minkowski did not bother to state this proof (that if the world were three-dimensional, none of the experiments captured in the relativity postulate, including the Michelson and Morley experiment, would be possible) explicitly; as a mathematicion he believed it was indeed self-evident.

sented by the horizontal line in the figure) of an observer at rest with respect to the body, is the body's proper length. The three-dimensional cross-section $P'P'$, resulting from the intersection of the body's worldtube and the space (represented by the inclined dashed line) of an observer at rest with respect to the second body (represented by the inclined worldtube), is the relativistically contracted length of the body measured by that observer (one should always keep in mind that the cross-section $P'P'$ only looks longer than $PP$ because a fact of the pseudo-Euclidean geometry of spacetime is represented on the Euclidean surface of the page).

Now assume that the worldtube of the body did not exist as a four-dimensional object and were merely an abstract geometrical construction. Then, what would exist would be a single three-dimensional body, represented by the proper cross-section $PP$, and both observers would measure the *same* three-dimensional body $PP$ of the *same* length. Therefore, not only would length contraction be *impossible*, but relativity of simultaneity would be also impossible since a spatially extended three-dimensional object is defined in terms of *simultaneity* – as all parts of a body taken *simultaneously* at a given moment.[5] Because both observers in relative motion would measure the same three-dimensional body (represented by the cross-section $PP$) they would share the *same* class of simultaneous events (therefore simultaneity would turn out to be absolute) in contradiction with relativity and with the experiments that confirmed the specific manifestations of relativity of simultaneity – length contraction and time dilation.

All experiments that confirmed time dilation and the twin paradox effect are also impossible in a three-dimensional world [19]. For example, it is an experimental fact, used every second by the GPS, that observers in relative motion have different times, which is impossible in a three-dimensional world [19].

I think the unprecedented situation in fundamental physics – ignoring the fact that the relativistic experiments and the theory of relativity itself are impossible in a three-dimensional world[6] – should be faced and addressed because this situation prevents a proper understanding of the physical meaning of general relativity as revealing that gravitational phenomena are nothing more than a manifestation of the curvature of spacetime; such a deep understanding of the nature of gravity may have important implications for the research on quantum gravity and on gravitational waves.

After Minkowski explained in his lecture *Space and Time* that the true reality is a four-dimensional world in which all ordinarily perceived three-

[5]The fact that an extended three-dimensional body is defined in terms of simultaneity confirms Minkowski's interpretation of the cross-sections $PP$ and $P'P'$ as two three-dimensional bodies – while measuring the *same* body, the two observers measure *two* three-dimensional bodies represented by the two cross-sections. This relativistic situation only looks paradoxical at first sight because what is meant by "the same body" is the body's worldtube; the cross-sections $PP$ and $P'P'$ represent the two three-dimensional bodies measured by the two observers.

[6]It appears to be a real problem in physics that some physicists regard issues such as the reality of spacetime as belonging to philosophy, which is *physics at its worst* - the issue of the dimensionality of the world is pure physics.

dimensional particles are a forever given web of worldlines, he outlined his ground-breaking idea of regarding physics as spacetime geometry [1, p. 112]:

> The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.

Then he started to implement his program by explaining that inertial motion is represented by a timelike *straight* worldline, after which he pointed out that [1, p. 115]:

> With appropriate setting of space and time the substance existing at any worldpoint can always be regarded as being at rest.

In this way he explained not only *why* the times of inertial observers are equivalent (their times can be chosen along their timelike worldlines and all straight timelike worldlines in spacetime are equivalent) but also the physical meaning of the relativity principle – the physical laws are the same for all inertial observers (inertial reference frames), i.e. all physical phenomena look exactly the same for all inertial observers, because every observer describes them in his own space (in which he is at rest) and uses his own time. For example the speed of light is the same for all observers because each observer measures it in its own space using his own time.

Then Minkowski explained that accelerated motion is represented by a *curved* or, more precisely, *deformed* worldline and noticed that "Especially the concept of *acceleration* acquires a sharply prominent character."

As Minkowski knew that a particle moving by inertia offers no resistance to its motion with constant velocity (which explains why inertial motion cannot be detected experimentally as Galileo first demonstrated), whereas the accelerated motion of a particle can be discovered experimentally since the particle *resists* its acceleration, he might have very probably linked the sharp physical distinction between inertial (non-resistant) and accelerated (resistant) motion with the sharp geometrical distinction between inertial and accelerated motion represented by straight and deformed (curved) worldlines, respectively.

The realization that an accelerated particle (which resists its acceleration) is a deformed worldtube in spacetime would have allowed Minkowski to notice two virtually obvious implications of this spacetime fact [19]:

- The acceleration of a particle is absolute not because it accelerates with respect to some absolute space, but because its worldtube is deformed, which is an absolute geometrical and physical fact.
- The resistance a particle offers to its acceleration (i.e. its inertia) originates from a four-dimensional stress in its *deformed* worldtube.[7] That is, the

[7]Note that the worldtube, and therefore spacetime itself, must be real for this to be possible. The very correspondence between the sharp physical and geometrical distinction of inertial and accelerated motion strongly (and independently) implies that spacetime is real.

inertial force with which the particle resists its acceleration turns out to be a static restoring force arising in the *deformed* worldtube of the accelerated particle. I guess Minkowski might have been particularly thrilled by this implication of his program to regard physics as spacetime geometry because inertia happens to be another manifestation of the fact that reality is a four-dimensional world.

To demonstrates the enormous potential of Minkowski's program of geometrizing physics let us assume that Minkowski had read Galileo's works, particularly Galileo's analysis demonstrating that heavy and light bodies fall at the *same* rate [20]. In this analysis Galileo virtually came to the conclusion that a falling body does not resist its fall [20]:

> But if you tie the hemp to the stone and allow them to fall freely from some height, do you believe that the hemp will press down upon the stone and thus accelerate its motion or do you think the motion will be retarded by a partial upward pressure? One always feels the pressure upon his shoulders when he prevents the motion of a load resting upon him; but if one descends just as rapidly as the load would fall how can it gravitate or press upon him? Do you not see that this would be the same as trying to strike a man with a lance when he is running away from you with a speed which is equal to, or even greater, than that with which you are following him? You must therefore conclude that, during free and natural fall, the small stone does not press upon the larger and consequently does not increase its weight as it does when at rest.

Then the path to the idea that gravitational phenomena are manifestations of the curvature of spacetime would have been open to Minkowski – the experimental fact that a falling particle accelerates (which means that its worldtube is curved), but offers no resistance to its acceleration (which means that its worldtube is not deformed) can be explained only if the worldtube of a falling particle is *both curved and not deformed*, which is impossible in the flat Minkowski spacetime where a curved worldtube is always deformed. Such a worldtube can exist only in a non-Euclidean spacetime whose geodesics are naturally curved due to the spacetime curvature, but are not deformed.

As for Minkowski spacetime (*die Welt*) was real, then it would not have been difficult for him (as a mathematician who listens to what the mathematical formalism tells him and is not affected by the appearance that gravitation is a physical interaction) to realize that gravitational phenomena are fully explained as manifestations of the non-Euclidean geometry of spacetime with no need to assume the existence of gravitational interaction. Indeed, particles fall toward the Earth's surface and planets orbit the Sun not due to a gravitational force or interaction, but because they move by inertia (nonresistantly); expressed in correct spacetime language, the falling particles and planets are geodesic worldlines (or rather worldtubes) in spacetime.

Minkowski would have easily explained the force acting on a particle on the Earth's surface, i.e. the particle's weight. The worldtube of a particle falling toward the ground is geodesic, which, in ordinary language, means that the particle moves by inertia (non-resistantly). When the particle lands on the ground it is prevented from moving by inertia and it resists the change of its inertial motion by exerting an inertial force on the ground. Like in flat spacetime the inertial force originates from the *deformed* worldtube of the particle which is at rest on the ground.[8] So the weight of the particle that has been traditionally called gravitational force would turn out to be inertial force, which naturally explains the observed equivalence of inertial and gravitational forces. While the particle is on the ground its worldtube is deformed (due to the curvature of spacetime), which means that the particle is being constantly subjected to a curved-spacetime acceleration (keep in mind that acceleration means deformed worldtube!); the particle resists its acceleration through the inertial force and the measure of the resistance the particle offers to its acceleration is its inertial mass, which traditionally has been called (passive) gravitational mass. This fact naturally explains the equivalence between a particle's inertial and gravitational masses, which turned out to be the same thing.

In this way, Minkowski would have again *explained* one more set of experimental facts which Einstein merely *postulated* – Einstein "explained" these experimental facts by his equivalence postulate. So Minkowski would have explained Einstein's equivalence postulate exactly like he explained Einstein's relativity postulate.

# 3 Second scenario

Now imagine that after his lecture *Space and Time* Minkowski found a very challenging mathematical problem and did not compete with Einstein for the creation of the modern theory of gravitation. But when Einstein linked gravitation with the geometry of spacetime Minkowski regretted his change of research interests and started to study intensely general relativity and its implications.

As a mathematician Minkowski would be greatly impressed by the genius of his former student Einstein for linking gravitation with the geometry of spacetime and by the elegent mathematical formalism developed by Einstein with the help of another former student (and a friend of Einstein) – Marcel Grossmann. At the same time Minkowski would be appalled by Einstein's inability to trust the mathematical formalism of his general relativity and to try to smuggle into the theory the apparently self-evident notions of gravitational interaction and gravitational energy.

Minkowski would see Einstein's general relativity as a triumph of his program of geometrizing physics and would reformulate, or rather properly interpret, general relativity by pointing out that:

[8]Note again that Minkowski would have explained this fact only because he regarded spacetime as real – a fact deduced from all failed *experiments* designed to detect absolute uniform motion.

- The new theory of gravitation demonstrates that gravitational phenomena are in fact nothing more than manifestations of the non-Euclidean geometry of spacetime.
- General relativity *itself* demonstrates that gravitational phenomena are *fully* explained by the non-Euclidean geometry of spacetime and are not caused by gravitational interaction – particles falling toward the Earth and planets orbiting the Sun *all move by inertia* and inertia by its very nature presupposes *no interaction.* In the correct spacetime language the falling particles' worldlines and the planets' worldlines are geodesics which represent inertial (i.e. *non-resistant*[9]) motion.
- There is no gravitational field and no gravitational force in Nature – the weight of a particle on the Earth's surface which has always, before the advent of general relativity, been regarded as a gravitational force (caused by the Earth's gravitational field) is, according to a proper understanding of the mathematical formalism of general relativity (and as Minkowski would have found as we saw in the first scenario), *inertial* force.[10]
- As a mathematician Minkowski would point out that the mathematical formalism of general relativity provides additional proof that gravitational phenomena are not caused by gravitational *interaction* – the mathematical formalism of general relativity *itself* refuses to yield a proper (tensorial) expression for gravitational energy and momentum, which demonstrates that these are not present in the physical world. Moreover, the fact that "in relativity there is no such thing as the force of gravity" [21] implies that there is no gravitational energy either since such energy is defined as the work done by gravitational forces. Whether or not gravitational energy is regarded as local does not affect the very definition of energy.
- Minkowski's approach to understanding gravitational phenomena would help him identify the major open question in gravitational physics – *how*

[9] It is an experimental fact that particles falling toward the Earth's surface *do not resist* their fall – a falling accelerometer, for example, reads zero resistance (i.e. zero acceleration; the observed *apparent* acceleration of the accelerometer is caused by the spacetime curvature caused by the Earth). The experimental fact that particles do not resist their fall (i.e. their apparent acceleration) means that they move by inertia and therefore no gravitational force is causing their fall. It should be emphasized that a gravitational force would be required to accelerate particles downwards *if and only if* the particles *resisted* their acceleration, because *only then* a gravitational force would be needed to *overcome* that resistance.

[10] Einstein believed (as the quote in the Introduction reveals) that the geometrization of gravitation is nothing more than a mathematical representation of real gravitational interaction with real gravitational force and energy. Therefore, it seems Einstein had misinterpreted his "happiest thought" – he might have believed that the gravitational force acting on a particle, causing its fall, is somehow compensated by the inertial force with which the particle resists its downward acceleration (in line with his equivalence principle). However, that would not explain his "happiest thought" that a falling person "will not feel his own weight," because if there were a gravitational force acting on the person, *his fall would not be non-resistant* – his body will *resist* the gravitational force which accelerates it downwards (exactly like a particle accelerated by a force in open space resists its acceleration); the very physical meaning of the inertial force is that it is a *resistance* force, with which a particle resists its acceleration.

*matter changes the geometry of spacetime.*

# 4 Instead of Conclusion

*Gravitation as a separate agency becomes unnecessary*
Arthur S. Eddington [22]

*An electromagnetic field is a "thing;" gravitational field is not, Einstein's theory having shown that it is nothing more than the manifestation of the metric*
Arthur S. Eddington [23]

Despite that taken at face value general relativity fully explains gravitational phenomena without assuming that there exists gravitational interaction, there have been continuing attempts (initiated by Einstein) to smuggle the concept of gravitational interaction into the framework and mathematical formalism of general relativity.

Despite the arguments Minkowski would have pointed out (listed above), the prevailing view among relativists is that there exists indirect astrophysical evidence for the existence of gravitational energy – coming from the interpretation of the decrease of the orbital period of the binary pulsar system PSR 1913+16 discovered by Hulse and Taylor in 1974 [24] (and other such systems discovered after that), which is believed to be caused by the loss of energy due to gravitational waves emitted by the system (which carry away gravitational energy).

This interpretation that gravitational waves carry gravitational energy should be carefully scrutinized (especially after the recent detection of gravitational waves) by taking into account the arguments against the existence of gravitational energy and momentum and especially the fact that there does not exist a rigorous (analytic, proper general-relativistic) solution for the two body problem in general relativity. I think the present interpretation of the decrease of the orbital period of binary systems contradicts general relativity, particularly the geodesic hypothesis (geodesics represent inertial motion) and the experimental evidence (falling particles do not resist their fall) which confirmed it, because by the geodesic hypothesis the neutron stars, whose worldlines had been regarded as exact geodesics (since the stars had been modelled dynamically as a pair of orbiting *point* masses by Hulse and Taylor), *move by inertia without losing energy since the very essence of inertial motion is motion with no loss of energy.* For this reason no energy can be carried away by the gravitational waves emitted by the binary pulsar system. Let me stress it as strongly as possible: the geodesic hypothesis (confirmed by experiment) and the assertion that bodies, whose worldlines are geodesics, emit gravitational energy (carried away by gravitational waves), cannot be both correct.

In fact, it is the very assumption that the binary system emits gravitational waves which contradicts general relativity in the first place, because motion by inertia does not generate gravitational waves in general relativity. The inspiralling neutron stars in the binary system were modelled as *point* masses and therefore their worldlines are exact geodesics, which means that the stars move by inertia and no emission of gravitational radiation is involved; if the stars were modelled as extended bodies, then and only then they would be subject to tidal effects and energy would be involved, but that energy would be negligibly small (see next paragraph) and would not be gravitational (see the explanation of the origin and nature of energy in the sticky bead argument below). So, the assertion that the inspiralling neutron stars in the binary system PSR 1913+16 generate gravitational waves is incorrect because it contradicts general relativity.

Gravitational waves are emitted only when the stars' timelike worldlines are not geodesic,[11] that is, when the stars are subject to an absolute (curved-spacetime) acceleration (associated with the absolute feature that a worldline is not geodesic), not a relative (apparent) acceleration between the stars caused by the geodesic deviation of their worldlines. For example, in general relativity the stars are subject to an absolute acceleration when they collide (because their worldlines are no longer geodesic); therefore gravitational waves – carrying no gravitational energy-momentum – are emitted only when the stars of a binary system collide and merge into one, that is, "Inspiral gravitational waves are generated during the end-of-life stage of binary systems where the two objects merge into one" [25].

Let me repeat it: when the stars follow their orbits in the binary system, they do not emit gravitational waves since they move by inertia according to general relativity (their worldlines are geodesic and no absolute acceleration is involved); even if the stars were modelled as extended bodies, the worldlines of the stars' constituents would not be geodesic (but slightly deviated from the geodesic shape) which will cause tidal friction in the stars, but the gravitational waves generated by the very small absolute accelerations of the stars' constituents will be negligibly weak compared to the gravitational waves believed to be emitted from the spiralling stars of the binary system (that belief arises from using not the correct general-relativistic notion of acceleration ($a^\mu = d^2x^\mu/d\tau^2 + \Gamma^\mu_{\alpha\beta}(dx^\alpha/d\tau)(dx^\beta/d\tau)$), but the Newtonian one).

The famous sticky bead argument has been regarded as a decisive argu-

[11] The original prediction of gravitational wave emission, obtained by Einstein (*Berlin. Sitzungsberichte*, 1916, p. 688; 1918, p. 154), correctly identified the source of such waves – a spinning rod, or any rotating material bound together by cohesive force. None of the particles of such rotating material (except the centre of rotation) are geodesic worldlines in spacetime and, naturally, such particles will emit gravitational waves. This is not the case with double stars; as the stars are modelled as point masses, their worldliness are exact geodesics (which means that the stars are regarded as moving by inertia) and no gravitational waves are emitted. If the stars are regarded as extended bodies their worldtubes will still be geodesic, but their motion will not be entirely non-resistant, because of the tidal friction within the stars (caused by the fact that the worldlines of the stars' constituents are not congruent due to geodesic deviation).

ment in the debate on whether or not gravitational waves transmit gravitational energy because it has been perceived to demonstrate that gravitational waves do carry gravitational energy which was converted through friction into heat energy [26]:

> The thought experiment was first described by Feynman (under the pseudonym "Mr. Smith") in 1957, at a conference at Chapel Hill, North Carolina. His insight was that a passing gravitational wave should, in principle, cause a bead which is free to slide along a stick to move back and forth, when the stick is held transversely to the wave's direction of propagation. The wave generates tidal forces about the midpoint of the stick. These produce alternating, longitudinal tensile and compressive stresses in the material of the stick; but the bead, being free to slide, moves along the stick in response to the tidal forces. If contact between the bead and stick is 'sticky,' then heating of both parts will occur due to friction. This heating, said Feynman, showed that the wave did indeed impart energy to the bead and rod system, so it must indeed transport energy.

However, a careful examination of this argument reveals that kinetic, not gravitational, energy is converted into heat because a gravitational wave changes the shape of the geodesic worldline of the bead (and of the stick) and the stick prevents the bead from following its changed geodesic worldline, i.e., prevents the bead from moving by inertia; as a result the bead resists and exerts an *inertial* force on the stick (exactly like when a particle away from gravitating masses moving by inertia is prevented from its inertial motion, it exerts an inertial force on the obstacle and the kinetic energy of the particle is converted into heat).

It appears more adequate if one talks about *inertial*, not kinetic, energy, because what is converted into heat (as in the sticky bead argument) is the energy corresponding to the work done by the inertial force (and it turns out that that energy, originating from the inertial force, is equal to the kinetic energy [27]). The need to talk about the adequate inertial, not kinetic, energy is clearly seen in the explanation of the sticky bead argument above – initially (before the arrival of the gravitational wave) the bead is at rest and does not possess kinetic energy; when the gravitational wave arrives, the bead starts to move but by inertia (non-resistantly) since the shape of its geodesic worldline is changed by the wave into another geodesic worldline (which means that the bead goes from one inertial state – rest – into another inertial state, i.e., without any transfer of energy from the gravitational wave; transferring energy to the bead would occur if and only if the gravitational wave changed the state of the bead from inertial to non-inertial), and when the stick tries to prevent the bead from moving by inertia, the bead resists and exerts an inertial force on the stick (that is why, what converts into heat through friction is inertial energy).

Finally, it is a fact in the rigorous structure of general relativity that gravitational waves do not carry gravitational energy,[12] which, however, had been inexplicably ignored, despite that Eddington explained it clearly in his comprehensive treatise on the mathematical foundations of general relativity *The Mathematical Theory of Relativity* [23, p. 260]: "The gravitational waves constitute a genuine disturbance of space-time, but their energy, represented by the pseudo-tensor $t_\mu^\nu$, is regarded as an analytical fiction" (it cannot be regarded as an energy of any kind for the well-known reason that "It is not a tensor-density and it can be made to vanish at any point by suitably choosing the coordinates; we do not associate it with any absolute feature of world-structure," *ibid*, p. 136).

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[12] An immediate and misleading reaction "A wave that carries no energy?!" should be resisted, because it is from the old times of three-dimensional thinking – assuming that a wave really travels in the external world. There is no such thing as a propagating wave in spacetime – what is there is a spacetime region whose "wavelike" geometry is interpreted in three-dimensional language as a wave which propagates in space (exactly like a timelike worldline is interpreted in three-dimensional language as a particle which moves in space); also, keep in mind that there is no such thing as space in the external world, because spacetime is not divided into a space and a time.

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