1 Introduction

This is not a typical book on relativity. It puts the emphasis on conceptual questions that lie beyond the scope of most physics books on this subject. The idea of such a book started to emerge more than twenty five years ago when I was struggling to *understand the meaning* of the consequences of special and general relativity. At that time I failed to find any physics books on relativity which addressed questions that looked so obvious to me. Here are three examples of such questions:

- It is stated in all books on special relativity that uniform motion is relative but no need has been seen to explain *why* absolute uniform motion does not exist. Answering this question is crucial for a genuine understanding of special relativity as the following apparent paradox demonstrates. Our common sense tells us that if a body moves *in* space it moves *with respect to* space. And indeed if we consider different examples of something moving in something else, it does appear that the expressions ‘moving in’ and ‘moving with respect to’ are equivalent. However, according to relativity such a conclusion is wrong since it is implicitly based on the idea of absolute motion. Therefore in relativity it is still correct to say that an object moves *in* space but not *with respect to* space. It is precisely here that the question of the non-existence of absolute uniform motion should be addressed in order to explain the profound depth of what lies behind the seemingly innocent difference between the two expressions.
- Another important issue that needs special attention is the physical *meaning* of the relativity of simultaneity. Logically, it comes after the question of absolute motion and can be approached differently depending on whether it is discussed in a physics or philosophy of physics class. In a physics class on relativity, my favourite problem for starting the analysis of what the physical meaning of the relativity of simultaneity is is the following:

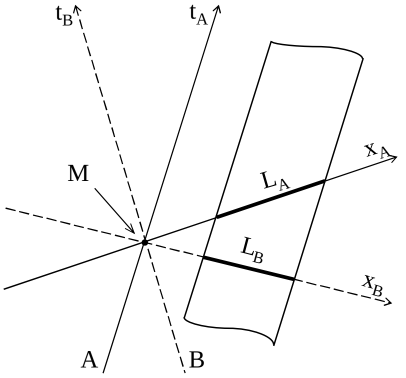
> An inertial reference frame S' moves with respect to another inertial reference frame S in the positive x direction of S. The clocks in S and S' are synchronized at the instant $t = t' = 0$ when the coordinate origins O and O' of the two frames coincide. At this moment a light wave is emitted from the point $O \equiv O'$. After time t it is observed in S that the light wave is spherical with a radius $r = ct$ and is described by the equation $r^2 = x^2 + y^2 + z^2$, which means that the center of the light sphere as determined in S is at O. Find the shape of the light wavefront in S' at time t'. Is it also a sphere whose center is at O'? If so, does this lead to a paradox? If not, does this lead to a contradiction with the principle of relativity?

The relativity principle requires all physical phenomena to look the same in all inertial reference frames. Therefore an observer in S' should determine that the wavefront of the propagating light signal is also a sphere whose center is at O'. This conclusion is confirmed by the Lorentz transformations. But our everyday experience tells us that there must be something totally wrong here – the center of the *same* light wave cannot be at two *different* places (at O and O' which may be thousands of kilometers apart). The standard explanation of this apparent paradox is the following: the wavefront of the propagating light sphere constitutes a set of *simultaneous* events and since according to relativity simultaneity is relative, the observers in S and S' have different sets of simultaneous events and consequently *different* light spheres. This is a correct explanation. But are you satisfied? I doubt it. This explanation is conceptually incomplete since it merely shifts the paradox from the specific case of light propagation to the relativity of simultaneity itself. What remains unexplained is *why* the two observers in S and S', who are in relative motion, have *different* sets of simultaneous events and therefore different light spheres (one centered at O and the other at O') given the fact that the two spheres originated from a *single* light signal. If the *physical meaning* of the relativity of simultaneity is explained conceptually then this apparent paradox will be explained as well.

- The above two questions as well as the question of the physical meaning of length contraction, time dilation, and the twin paradox all lead to the same major issue – how spacetime should be understood. Almost a century after Hermann Minkowski united space and time into an indivisible four-dimensional entity – now called Minkowski spacetime – the question “What is the nature of

> spacetime?" still remains open. In my view, this question should be addressed, not only in papers and books on the philosophy of spacetime, but in every physics book or university physics course on relativity. So far this has not been done, perhaps because most physicists seem to believe that their job is to make predictions which can be experimentally tested and that they need not bother about conceptual questions such as the following: Is Minkowski spacetime nothing more than a four-dimensional *mathematical* space which represents an evolving-in-time three-dimensional world or a mathematical model of a four-dimensional world with time entirely given as the fourth dimension? However, such conceptual questions cannot be avoided since the ultimate intellectual goal of all sciences, including physics, is to *understand* the world we live in.

In fact, even apart from pure intellectual curiosity, physicists themselves do need to address issues dealing with the interpretation of relativity if they want to offer some *explanation* of relativistic effects, which can make their mathematical description more transparent. Take for example length contraction as depicted in the figure below. Two inertial observers A and B in relative motion are represented by their worldlines (the lines of their entire lives in time). A meter stick is at rest in A's reference frame and is represented by its worldtube (its entire history in time) in the spacetime diagram shown in the figure.



The length of the meter stick is measured by A and B at event M when the observers meet, i.e., at the moment they set their clocks to zero: $t^A = t^B = 0$. As any length measurement requires that both ends of the meter stick be measured at the *same* time, and since A and B have different sets of simultaneous events, it follows that what A and B regard as their meter stick is, in fact, a *different* three-dimensional cross-section of the meter stick's worldtube. As the x axes of A and B intersect the worldtube at different angles, the two cross-sections L_A

and L_B are of different lengths, and this *explains* why A and B measure different lengths for the meter stick. The exact relation between the two lengths is obtained by the Lorentz transformations, which do show that $L_B < L_A$.

It is here that physicists cannot avoid the conceptual question of the nature of the meter stick's worldtube: Is the worldtube nothing more than just a graphical representation of the length contraction or a real four-dimensional object containing the whole history in time of the three-dimensional meter stick? It is clear from the spacetime diagram that, if we reject the reality of the worldtube of the meter stick, then A and B cannot have different cross-sections since only A's meter stick of length L_A would exist. This means that the same meter stick of the *same* length L_A would exist for B as well and no length contraction would be possible. Therefore the very existence of the relativistic length contraction seems to imply the reality of the meter stick's worldtube. This in turn implies the reality of Minkowski spacetime, since four-dimensional objects exist in a four-dimensional world.

Most books on relativity do not use spacetime diagrams specifically in the discussions of kinematic relativistic effects and do not face the immediate need to address the issue of the nature of Minkowski spacetime. Once obtained through the Lorentz transformations, these effects are not usually explained any further. In my view, such an approach is unsatisfactory for two reasons. Most importantly, physics is much more than its mathematical formalism and therefore everything should be done to provide a physical explanation of the results obtained through the Lorentz transformations. Secondly, if relativists themselves make no effort to shed some light on the meaning of the relativistic effects, different accounts start to emerge which in many cases are inconsistent with relativity itself.

One of the main reasons for writing this book is to address the issue of the physical meaning of the relativistic effects and the nature of spacetime by analyzing what the mathematical formalism of relativity is telling us. More specifically this is done:

- by carrying out an analysis of the idea of absolute motion starting from Aristotle's view on motion,
- by explicitly addressing the question of existence and dimensionality of the objects (rulers, clocks, twins, etc.) involved in the relativistic effects.

Part One entitled *From Galileo to Minkowski* starts with a chapter on the idea of absolute motion and how it was brought to its logical end by Galileo's refutation of Aristotle's view on motion. Chapter 3 is devoted to exploring the internal logic of Galileo's principle of relativity. I will argue that special relativity, and more precisely its four-dimensional formulation given by Minkowski, is *logically* contained in Galileo's principle of relativity (with a single additional assumption – that the speed of light is finite, which was determined experimentally in Galileo's century). An important result of this chapter will be the non-trivial conclusion that the non-existence of absolute uniform motion implies that the world is four-dimensional (or, equivalently, if the world were three-dimensional, absolute uniform motion had to exist because, as we will see in Chap. 3, a *single* three-dimensional world implies that 'moving *in* space' is equivalent to 'moving *with respect to* space'). Further exploration of the consequences of Galileo's relativity principle leads to all kinematic relativistic effects which are derived in Chap. 4. These derivations demonstrate that the relativistic effects are merely manifestations of the four-dimensionality of the world, whose geometry is pseudo-Euclidean, since these effects have direct analogs in the ordinary three-dimensional Euclidean space. One of the objectives of Part One is to show that special relativity could realistically have been formulated significantly earlier.

Part Two entitled *On the Nature of Spacetime – Conceptual and Philosophical Issues* is the most provocative of the three parts of the book. But it had to be written since the issues raised by the theory of relativity have challenged our entire world view in an unprecedented way. Never before has a scientific theory called for such a drastic revision of concepts that we have hitherto regarded as self-evident, such as the existence of:

- objective change,
- objective flow of time,
- free will.

In my view, special relativity has posed perhaps the greatest intellectual challenge humankind has ever faced. In this situation the best way to take on the challenge is to deal directly with its very core – the question of the nature of spacetime – since this question *logically precedes* the questions of change, flow of time, and free will. As we will see in Chap. 5, these issues crucially depend on what the dimensionality of the world is, which demonstrates that they are indeed preceded by the issue of the nature of spacetime.

For this reason the first chapter of Part Two (Chap. 5) examines the issue of the nature of Minkowski spacetime and argues that it is special relativity *alone* and the experimental evidence that confirms its predictions that can resolve this issue. This argument comes from the analysis carried out in the chapter which shows that special relativity is valid only in a four-dimensional world represented by Minkowski spacetime. Otherwise, if the world were three-dimensional, none of the kinematic relativistic effects would be possible, provided that the existence of the physical objects involved in the relativistic effects is assumed to be absolute (frame-independent). The only way to preserve the three-dimensionality of the world is to relativize existence. However, even this extreme step contradicts relativity itself and more specifically the twin paradox effect.

The profound implications of relativity (and its requirement that the world be four-dimensional) for a number of fundamental issues such as conventionality of simultaneity, temporal becoming, flow of time, free will, and even consciousness are also discussed in Chap. 5. It is shown that, in the four-dimensional Minkowski world:

- the definition of simultaneity is necessarily conventional,
- there are no objective becoming and time flow,
- there is no free will,
- the concept of consciousness (implicitly defined by Hermann Weyl [1] as an entity which makes us aware of ourselves and the world only at the moment ‘now’ of our proper time) is needed to reconcile the major consequence of special relativity that external reality is a timelessly existing four-dimensional world with the fact from our experience that we realize ourselves and the world only at the present moment.

It is these conclusions that constitute the intellectual challenge mentioned above. The most tempting way out of it is to declare them absurd or undoubtedly wrong. That is fine, if such a declaration is backed up by arguments demonstrating why those conclusions are wrong. A way to avoid facing the challenge is to subscribe to the view that we should accept the theory of relativity but should make no metaphysical pronouncement regarding the nature of spacetime. Such a view, however, completely ignores the fact that an analysis of the consequences of special relativity clearly shows that the challenge is there.

There exist two other approaches which try to avoid the challenge posed by special relativity. They purport to show that we should not bother about metaphysical conclusions drawn from special relativity for two reasons. According to the first approach the fact that relativity describes the world as four-dimensional and deterministic should not be taken as the whole truth since quantum mechanics, quantum gravity, and other modern physical theories are telling us different stories. Leaving aside the fact that quantum gravity and some of the modern physical theories are not yet accepted theories, Chap. 6 will make use of the results of Chap. 5 that it is the *experimental evidence* confirming the consequences of special relativity that contradicts the three-dimensionalist view. It would be really another story if the experimental evidence confirming the predictions of quantum mechanics contradicted the four-dimensionalist view. But this is not the case. Chap. 6 will present two arguments which demonstrate that quantum mechanics has nothing to say on the nature of spacetime.

Chapter 7 deals with the second approach according to which special relativity cannot tell us anything definite about the external world because, like any other theory, it may be disproved one day. We will see that this desperate attempt to avoid the challenge posed by relativity fails too. Again, this argument completely ignores the fact that it is the *experimental evidence* confirming the predictions of special relativity that contradicts the three-dimensionalist view. As experimental evidence cannot be disproved, any attack on the four-dimensionalist view should challenge the claim that experiment itself contradicts the accepted three-dimensionalist view. I will argue in this chapter that a scientific theory will never be disproved in its area of applicability where its predictions have been experimentally confirmed.

The main purpose of Part Two is to show convincingly that the challenge to our world view arising from special relativity – that the world is four-dimensional – is real. That is why it is only fair to face it now instead of leaving it for future generations.

Part Three entitled *Spacetime, Non-Inertial Reference Frames, and Inertia* further explores the consequences of the four-dimensionality of the world for physics itself. Chapter 8 starts by showing that relativity has resolved the debate over acceleration – whether it is absolute as Newton thought or relative as Leibnitz and Mach insisted. A body moving by inertia (with no acceleration) is represented in Minkowski spacetime by a straight worldtube; if the body accelerates, its worldtube is curved. Therefore, special relativity clearly shows that acceleration is absolute – there is an absolute difference between straight and curved worldtubes (and these worldtubes are, as argued in the book, not just convenient graphical representations but real four-dimensional objects).

The situation in general relativity is the same. The analog of a straight worldtube in a curved spacetime is a geodesic worldtube. A body moving by inertia (with *no curved spacetime acceleration*) is represented by a geodesic worldtube; if the body accelerates, its worldtube is deformed, i.e., deviated from its geodesic shape. Unlike relative velocity which cannot be discovered, an absolute acceleration should be detected experimentally. And indeed the propagation of light in a non-inertial reference frame, in which an accelerating body is at rest, turns out to be anisotropic – the average velocity of light depends on the body's acceleration. (The speed of light is c in all inertial reference frames in special relativity and in all local inertial reference frames in general relativity.) Most of Chap. 8 is devoted to the propagation of light in non-inertial reference frames – a topic that has received little attention up to now. The chapter ends with a discussion of the gravitational redshift effect and the Sagnac effect.

Chapter 9 shows that the potential and the electric field of a non-inertial charge can be calculated *directly* in the non-inertial reference frame in which the charge is at rest (without the need to transform the field from a comoving or local inertial frame) if the anisotropic velocity of light in that frame is taken into account. It is shown that the average anisotropic velocity of light in a non-inertial reference frame gives rise to a hitherto unnoticed anisotropic (Liénard–Wiechert-like) volume element which leads to the correct expressions for the potential and electric field of a charge in such a frame.

Chapter 10 addresses a natural question: If the deformed worldtube of an accelerating body is a real four-dimensional object, can the inertial force resisting the body's acceleration be regarded as originating from a four-dimensional stress in the body's worldtube which arises when the worldtube is deformed? It is argued in this chapter that inertia is another manifestation of the four-dimensionality of the world. Although the existence of inertia cannot be regarded as a definite proof of the reality of spacetime, it is shown in the chapter that, if the world is four-dimensional, inertia must exist.