Einstein-Podolsky-Rosen Argument, Bohr's Reply and in What Sense Quantum Mechanics is Fundamentally Incomplete

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https://vesselinpetkov.com/16Oct2025.html

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This is a second version of my talk. The first was more technical and I felt that at such conferences the talks should be rather on conceptual analyses and on the physical interpretation of the discussed phenomena.

OUTLINE

My talk is divided into three parts:

1. A summary of the 1935 paper by Einstein, Podolsky and Rosen
2. A summary of the 1935 response by Niels Bohr
3. A rather extended summary of an obvious, in my view, argument, which demonstrates that quantum mechanics is fundamentally incomplete (not in the sense of Einstein, Podolsky and Rosen)

In their 1935 paper Einstein, Podolsky and Rosen put forward an argument intended to demonstrate that quantum mechanics is incomplete since it does not assign simultaneous values for observables associated with the quantum object. The argument contained a thought experiment which led to the discovery of quantum entanglement — perhaps the most counter-intuitive and profound feature of quantum phenomena.

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

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In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

EPR, Phys. Rev., 1935

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

EPR, Phys. Rev., 1935

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It

Regarded not as a necessary, but merely as a sufficient, condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality.

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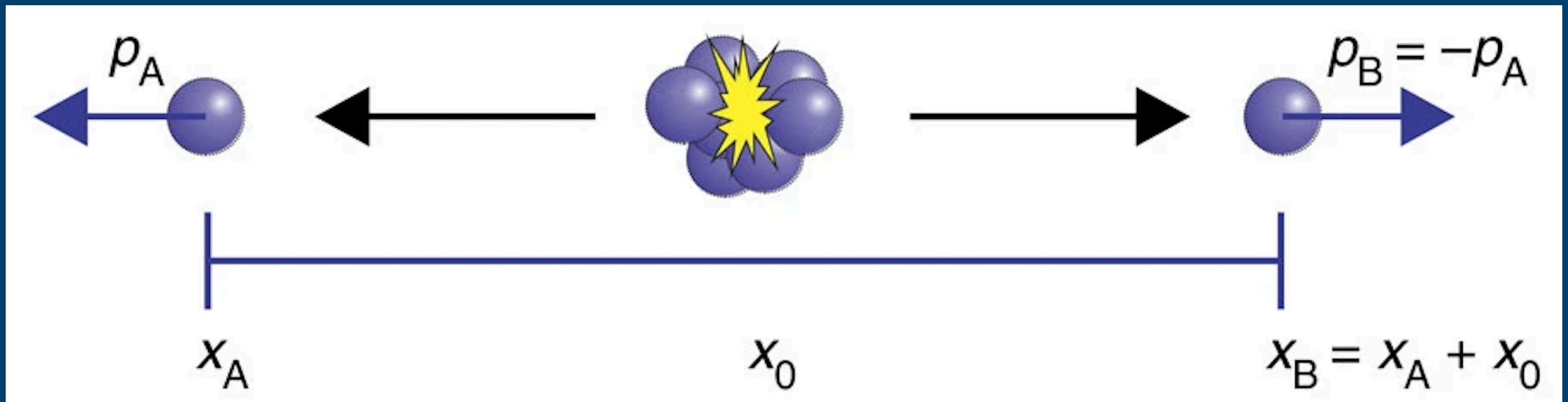


Satisfying the Einstein–Podolsky–Rosen criterion with massive particles https://www.nature.com/articles/ncomms9984

Returning now to the general case contemplated in Eqs. (7) and (8), we assume that ψ_k and φ_r are indeed eigenfunctions of some non-commuting operators P and Q, corresponding to the eigenvalues p_k and q_r, respectively. Thus, by measuring either A or B we are in a position to predict with certainty, and without in any way disturbing the second system, either the value of the quantity P (that is p_k) or the value of the quantity Q (that is q_r). In accordance with our criterion of reality, in the first case we must consider the quantity P as being an element of reality, in the second case the quantity Q is an element of reality. But, as we have seen, both wave functions ψ_k and φ_r belong to the same reality.

A, B - physically observable quantities

Previously we proved that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with noncommuting operators, can have simultaneous reality. Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.

EPR, Phys. Rev., 1935

One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*. On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes the reality of P and Q depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.

EPR, Phys. Rev., 1935

While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.

EPR, Phys. Rev., 1935

Bohr's response

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

IN a recent article[1] under the above title A. Einstein, B. Podolsky and N. Rosen have presented arguments which lead them to answer the question at issue in the negative. The trend of their argumentation, however, does not seem to me adequately to meet the actual situation with which we are faced in atomic physics. I shall therefore be glad to use this opportunity to explain in somewhat greater detail a general viewpoint, conveniently termed "complementarity," which I have indicated on various previous occasions,[2] and from which quantum mechanics within its scope would appear as a completely rational description of physical phenomena, such as we meet in atomic processes.

interaction with the system under investigation. According to their criterion the authors therefore want to ascribe an element of reality to each of the quantities represented by such variables. Since, moreover, it is a well-known feature of the present formalism of quantum mechanics that it is never possible, in the description of the state of a mechanical system, to attach definite values to both of two canonically conjugate variables, they consequently deem this formalism to be incomplete, and express the belief that a more satisfactory theory can be developed.

Such an argumentation, however, would hardly seem suited to affect the soundness of quantum-mechanical description, which is based

There were a number of reactions (published and in letters) to the EPR challenge. Let's start with Wolfgang Pauli's letter to Heisenberg:

Wolfgang Pauli reacted to the EPR paper in his habitual way, i.e., harshly. Already on June 15, 1935, he wrote to Heisenberg:

> *Einstein* once again commented publicly on quantum mechanics, this time in Physical Review on May 15 (together with Podolsky and Rosen – no good company, by the way). As is well known this is a catastrophe every time it happens. "For – he keenly concludes – that which must not, cannot be." (Morgenstern).

C. Kiefer, *Albert Einstein, Boris Podolsky, Nathan Rosen: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete* (Birkhäuser 2022), p. 65

In addition to the fact that such statements often best apply to their authors, Pauli is also known for his harsh criticism of any work of physicists with which Pauli disagrees:

It is not even wrong!

Unfortunately, such criticism of any attempts to ask deep questions about the physical meaning of counter-intuitive physical phenomena resulted in counter-productive fashion in theoretical physics summarized in the (in)famous expression

Shut up and calculate!

For two reasons, special attention will be given to Bohr's reply to EPR's work. On the one hand, it exemplarily shows the conceptual difficulties associated with the debate. On the other hand, it was historically the most important reaction to EPR's work, because many physicists considered Bohr to be *the* authority in the field of quantum theory that was not to be challenged. Many, if not most physicists followed Bohr without criticism and did not really bother to read EPR's and Bohr's original works. But as Mara Beller, in particular, has pointed out, Bohr's 'victory' over Einstein is but a legend and not based on facts [17, p. 151*f*]. According to Arthur Fine, the 'EPR paradox' is a paradox first and foremost if one adopts the Copenhagen interpretation of quantum theory [73, p. 4*f*]. This interpretation, we recall, had been formulated by Heisenberg and especially by Bohr, who thought of himself as this interpretation's creator, in the years after 1925, and had been dismissed by Einstein as "Heisenberg-Bohr tranquillising philosophy" as early as in 1928 in a letter to Schrödinger.[1] So let us first have a look at Bohr's paper.

C. Kiefer, *Albert Einstein, Boris Podolsky, Nathan Rosen: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete* (Birkhäuser 2022), p. 57

With respect to the underlined sentence above, it is worth mentioning Einstein's implicit criticism of those who blindly follow the authority:

"To punish me for my contempt for authority, fate made me an authority myself"

B. Hoffman, *Albert Einstein: Creator and Rebel* (Viking, New York 1972), p. 24

But first, let's have a look at this rather polite comment on the way Bohr presented his arguments in his response to the EPR paper:

Bohr's paper is not a prime example of clarity.[3] Mara Beller brought up the following quite amusing fact [16]. Most commentators refer to the reprint of Bohr's article in the collective volume dealing with the foundations of quantum theory edited by Wheeler and Zurek [164]. In this reprint, however, pages 700 and 699 were interchanged.[4] Nobody ever noticed. Indeed, when reading the paper in the wrong order of pages, one does not get a significantly different impression than when reading the original. The author seems to have known that his paper was incomprehensible. He later wrote about it: "Rereading these passages, I am deeply aware of the inefficiency of expression which must have made it very difficult to appreciate the trend of the argumentation [...]."[5] Is it still possible to extract the core messages from Bohr's paper?

C. Kiefer, *Albert Einstein, Boris Podolsky, Nathan Rosen: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete* (Birkhäuser 2022), p. 58

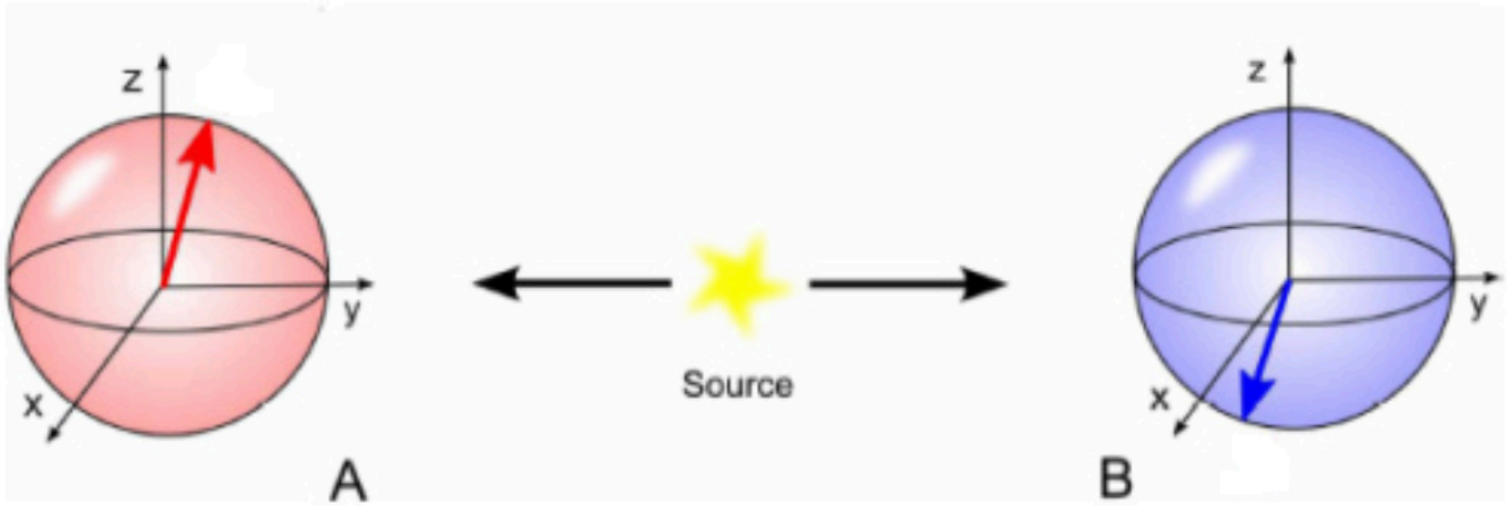
Already the introductory lines include two points that were essential to Bohr: EPR's 'criterion of physical reality' and the concept of complementarity, introduced by Bohr. According to the author, the application of complementarity will entail the completeness of the quantum mechanical description. In his paper, Bohr especially attacks EPR's criterion of reality, although it did not play a central role in the EPR paper as we have seen above.

Instead, he assigned a word to the situation in question – complementarity.

C. Kiefer, *Albert Einstein, Boris Podolsky, Nathan Rosen: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete* (Birkhäuser 2022), pp. 59-60

David Bohm's version of the Einstein Podolsky Rosen Argument

Think of the following situation: a particle with zero spin decays into two particles (A and B), each with 1/2-spin. Due to the fact that spin angular momentum must be conserved during the decay, if initially the total spin angular momentum was zero, then after the decaying process it *must* still be zero. Therefore, particles A and B have opposite spin.



Take as an example the dissociation of an exited hydrogen molecule into two hydrogen atoms. If the decaying mechanism does not change total angular momentum, then the spins on the hydrogen atoms will be anti-correlated. Whenever a measurement of the spin of A is found to be positive with respect of the *z*-axis (we shall note this state as $|\uparrow\rangle$), then, we could infer that the spin of the B particle must be negative ($|\downarrow\rangle$), and this is true *independent of the distance that separates the particles*. The spin of these particles are then *entangled*.

David Bohm's version of the Einstein Podolsky Rosen Argument

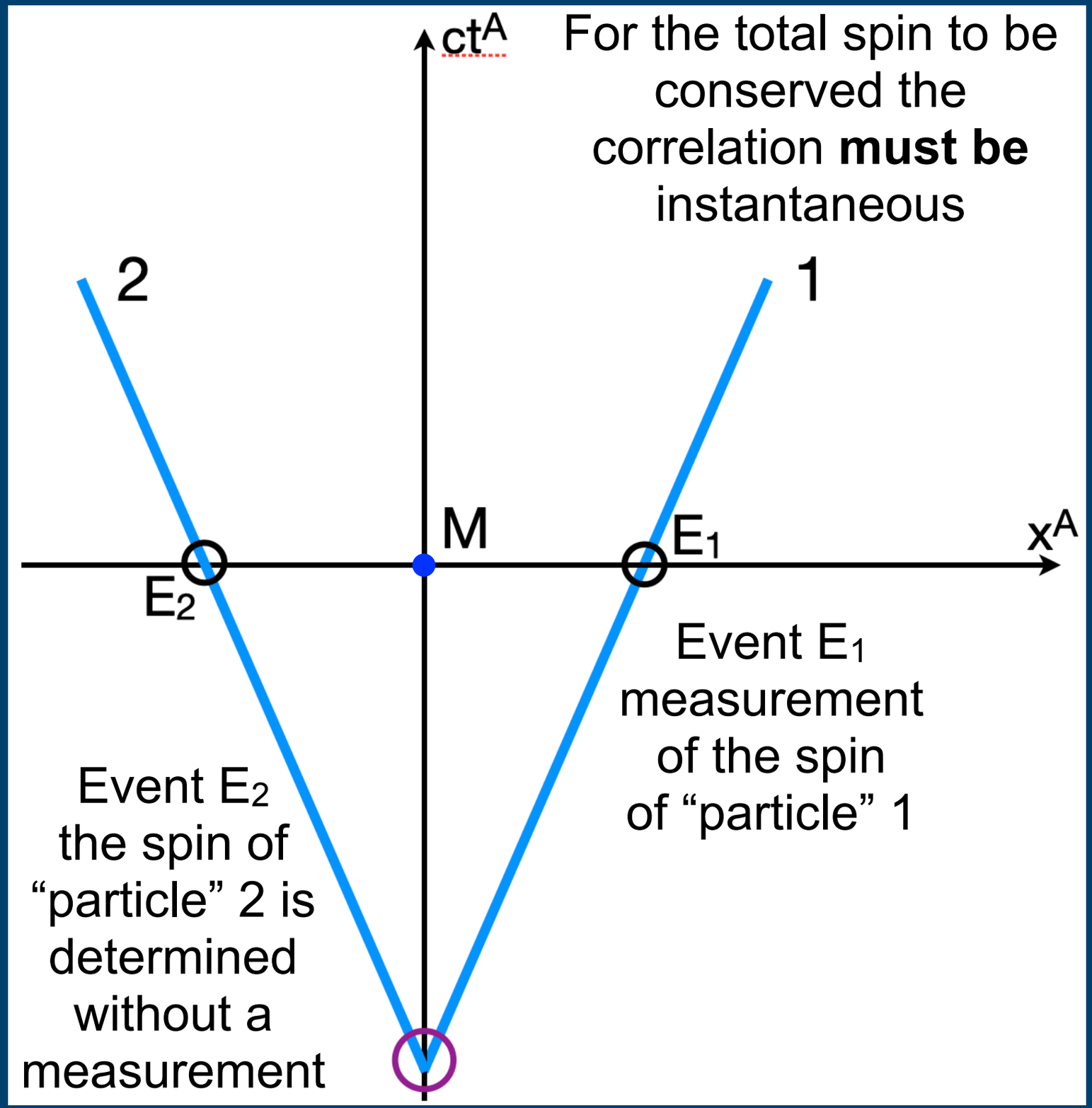
Now the molecule is thought to be fragmented into the two atoms by some dissociation process, and the atoms then separated to an arbitrarily large distance (in principle, their distance can be of astronomical magnitude). 'Arbitrarily large distance' means that an interaction between the two atoms is made impossible, in complete analogy to the two particles in EPR's original thought experiment. Note that the value of the total spin of the combined system is conserved.

The argument then continues as in the EPR paper. If one takes a measurement on the z-component of the first atom's spin, then one can deduce from it the z-component of the second atom's spin due to their anticorrelation; if one finds $+\hbar/2$ for atom 1, atom 2 must have $-\hbar/2$. Now the point is that one can of course take a measurement on atom 1's spin with respect to any other direction, for example the x-direction. If one finds $+\hbar/2$ for atom 1, atom 2's spin component in x-direction must be $-\hbar/2$. So just like EPR one can conclude that all spin orientations of atom 2 have simultaneous reality. The atom's spin orientations play the role of the conjugated quantities position and momentum. Just like those, the spins in different orientations do not commute and can therefore not be measured simultaneously, according to the laws of quantum mechanics. Thus, one can conclude the incompleteness of quantum mechanics from this version, too.

C. Kiefer, *Albert Einstein, Boris Podolsky, Nathan Rosen: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete* (2022), pp. 34-35

THE EPR ARGUMENT MAY LEAD TO SOME EYE-OPENING DIFFICULTIES

1. It is *c*orrelation not causation, but what is the physical meaning of this correlation?
2. And also the issue of conservation of the total spin (or the momentum involved in the original EPR argument).



For observer A events E_1 and E_2 are simultaneous.

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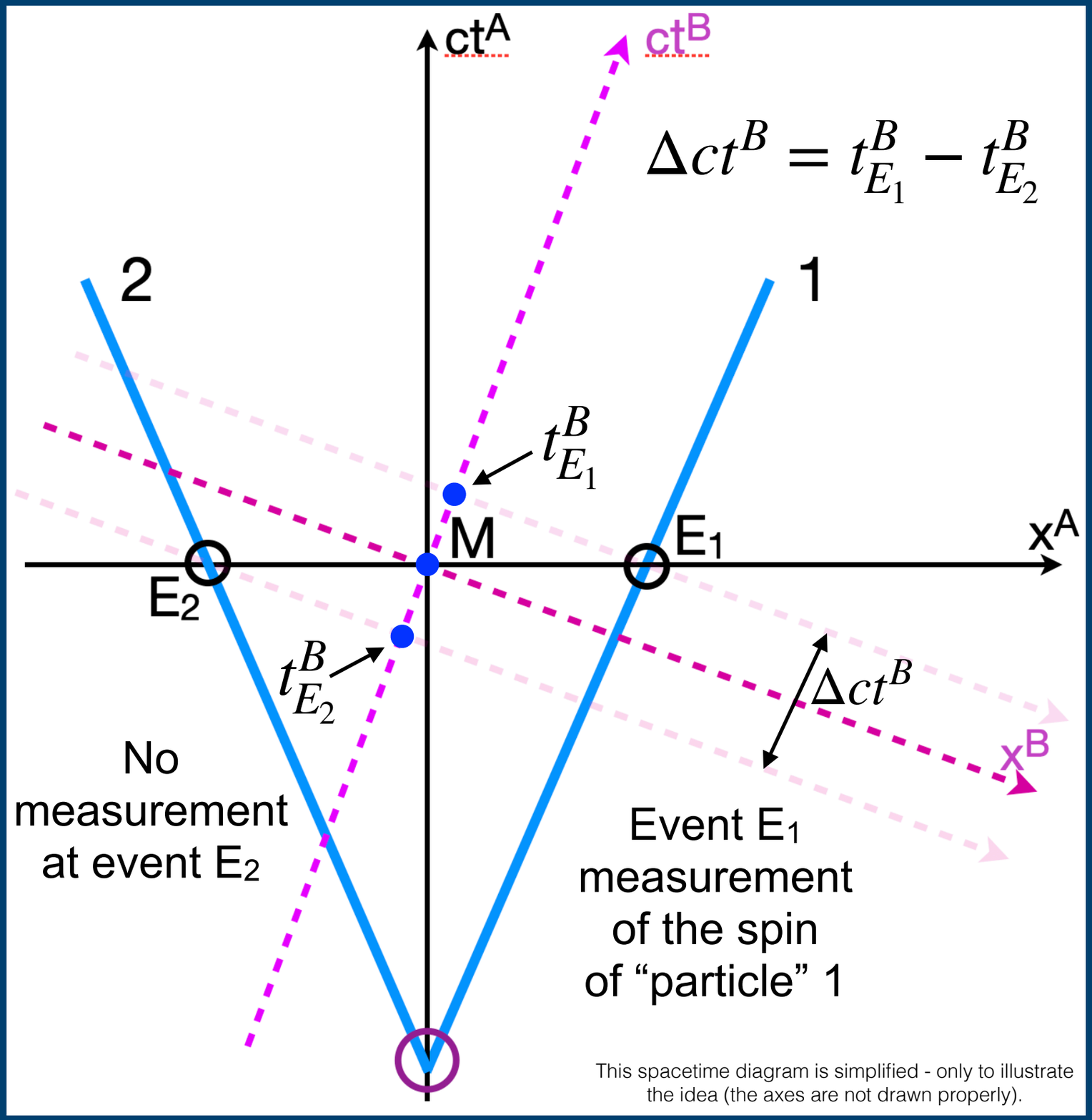

For observer A events E_1 and E_2 are simultaneous, but for observer B event E_2 occurs before event E_1, i.e. **before** the measurement! Inexplicably, this point has not been properly discussed.

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THE EPR ARGUMENT MAY LEAD TO SOME EYE-OPENING DIFFICULTIES

3. As we saw EPR defined “elements of physical reality” as:

If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding lo this physical quantity.

Amazingly, it seems the ultimate criterion of physical reality - the **experimental evidence** - appears to have been forgotten.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It

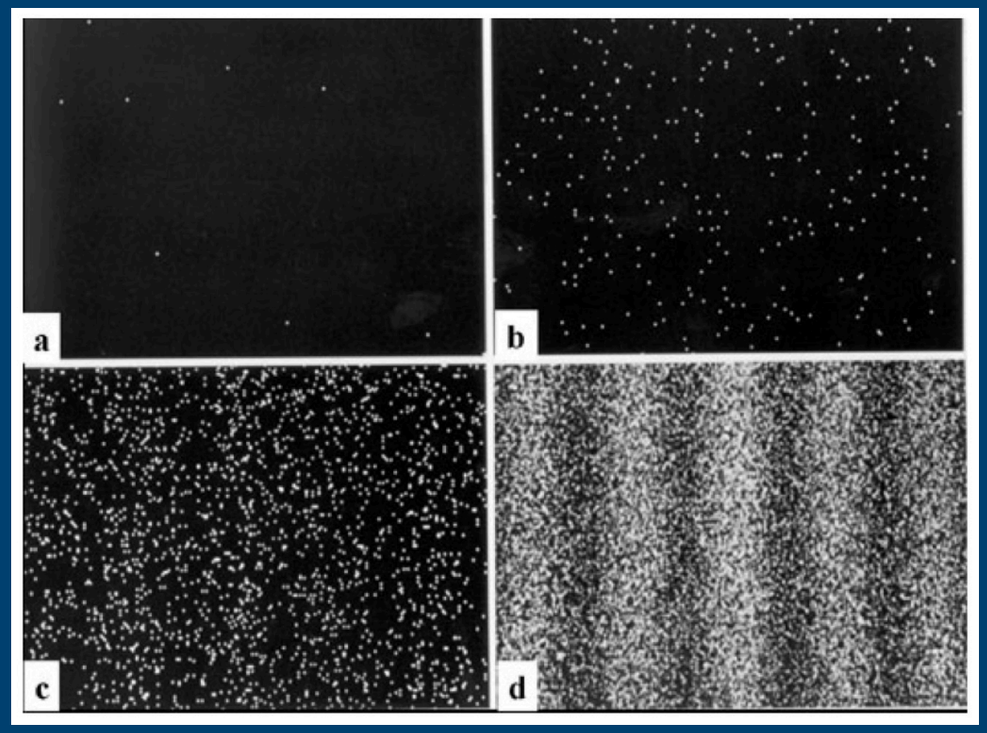
Regarded not as a necessary, but merely as a sufficient, condition of reality, this criterion is in agreement with classical as well as quantum-mechanical ideas of reality.

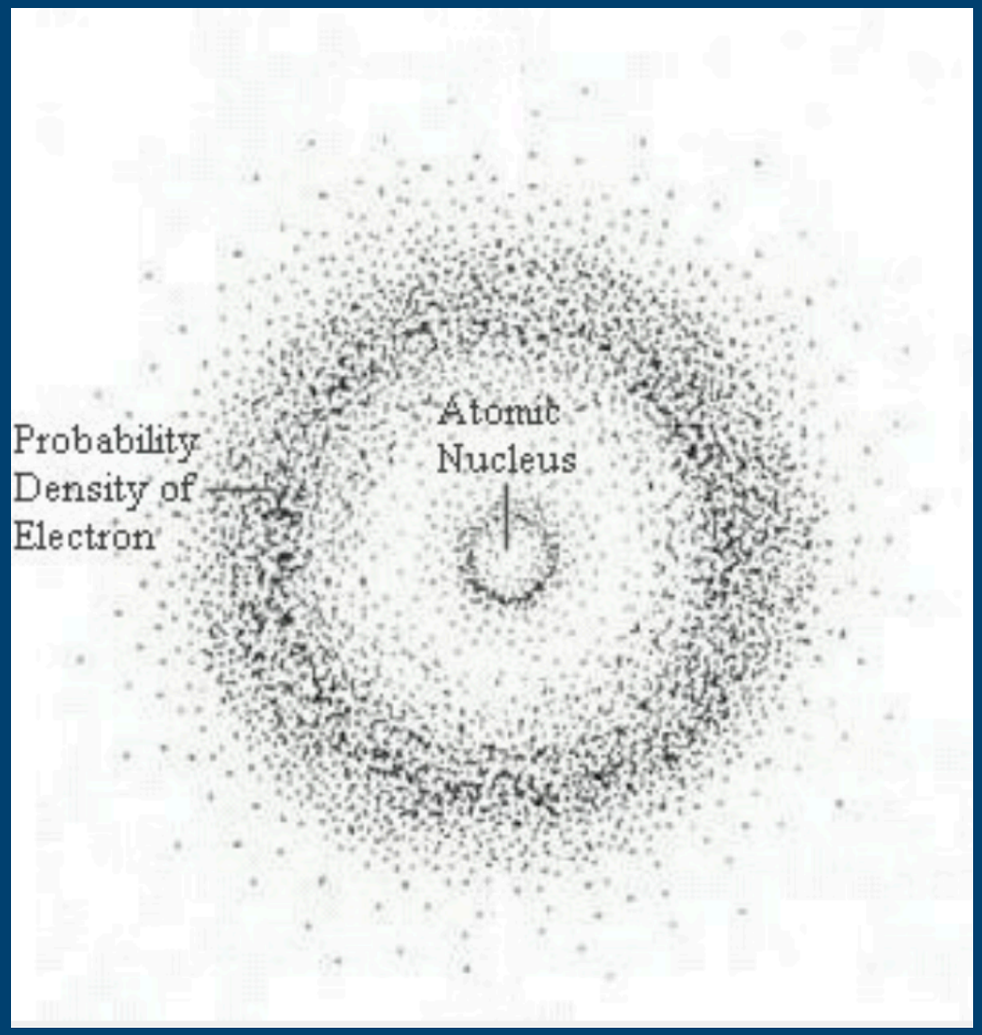
Let’s discuss two experimental facts which **prove** that a quantum object (an electron, for example) **exists everywhere where its wavefunction is different from zero**.

The double-slit experiments with single electrons - ***each*** electron passes through both slits; otherwise no interference pattern. This **proves** that equal probability for a single electron to pass through both slits means that the entity “electron” **actually** goes through the two slits.

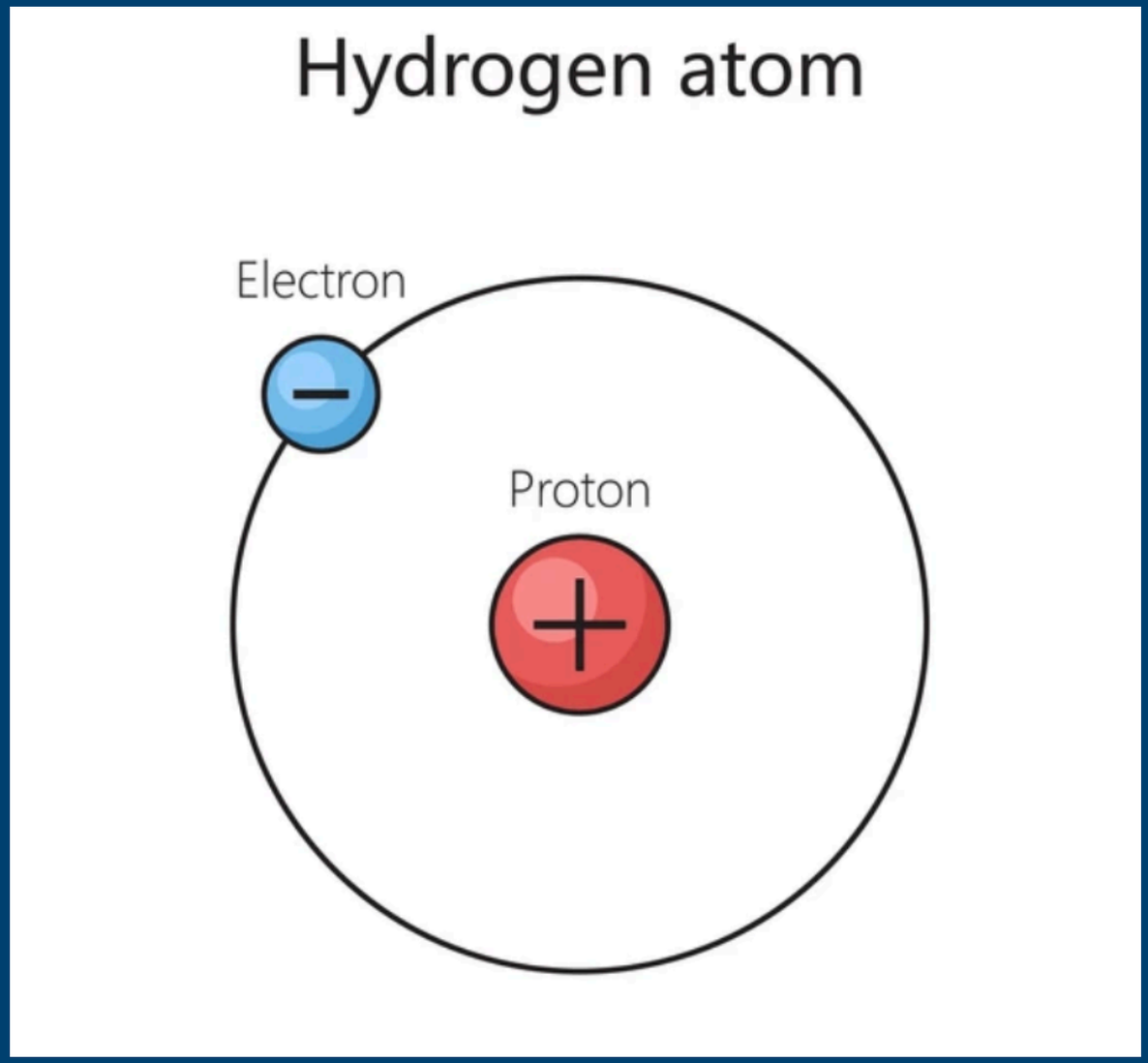
The experimental (and theoretical) fact that the electron in its ground (s-) state in the hydrogen atom does not possess a dipole moment **proves** that the electron **actually exists** on a spherical shell around the proton.

“**Proves**” because, otherwise, there would be a dipole moment.

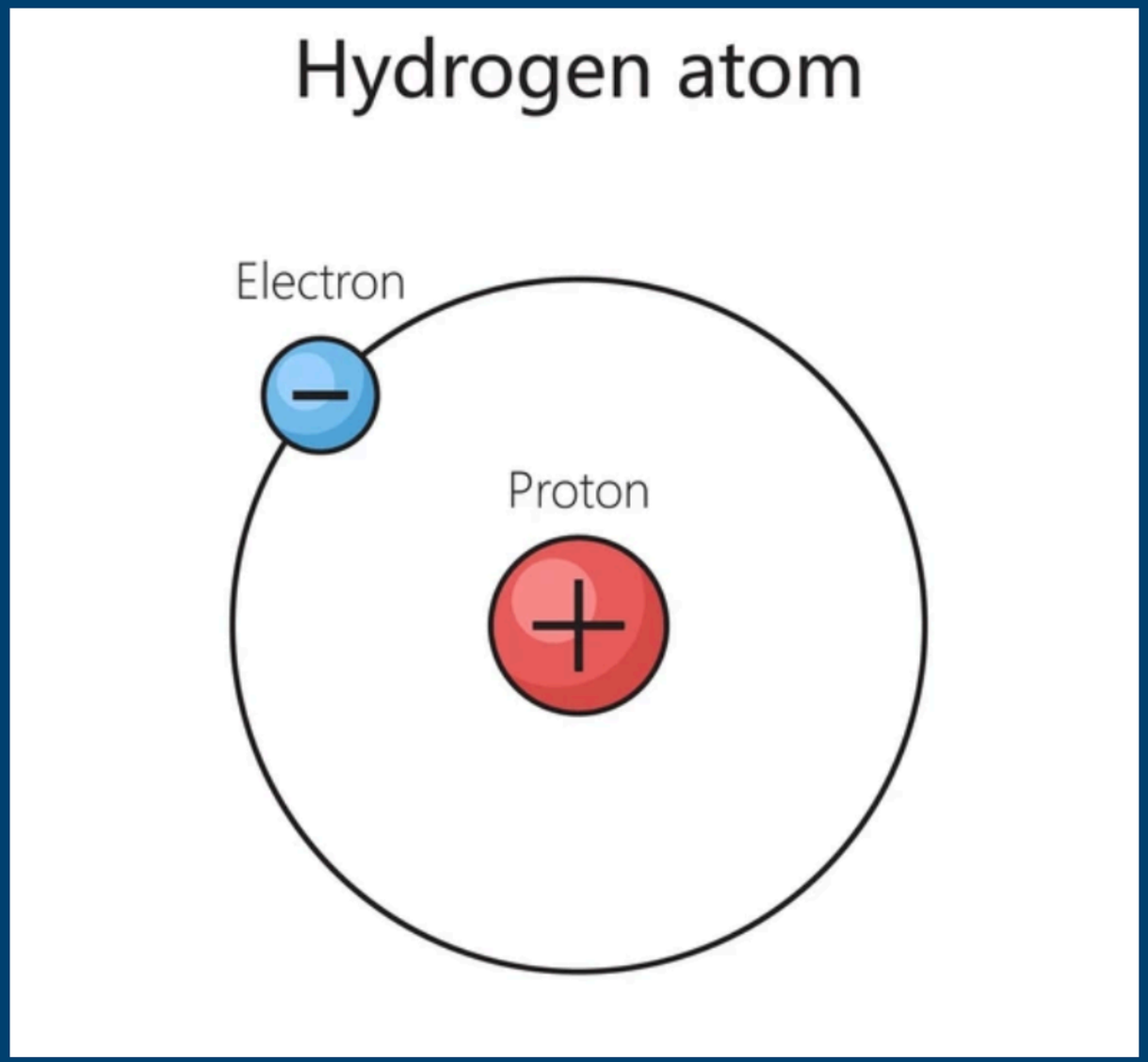




Quantum mechanics predicts that the electron in its ground (s-) state in the hydrogen atom can be found with a given probability around the nucleus (the proton). If the electron were regarded as a particle, i.e., as localized somewhere above the proton, then the hydrogen atom should possess a dipole moment in its ground state. Both quantum mechanics and experiment show that this is not the case.



One may picture the electron as so rapidly orbiting the proton that what is experimentally measured is the average value of the dipole moment over the measurement time. And since there is a spherical symmetry in the ground state, all dipole moments cancel out exactly – the average value is zero.



To verify this hypothesis Erwin Madelung calculated the orbital velocity of the electron that would ensure that all dipole moments during the measurement cancel out. It turned out that the electron orbital velocity should be ***several orders of magnitude greater than the velocity of light***.

This proves that the electron charge should be somehow **actually** distributed around the proton. In other words, when not measured, **the electron should exist everywhere where the probability of finding it is different from zero**. Such a requirement resolves the problem about the lack of a dipole moment of the hydrogen atom in s-state.

So, in quantum mechanics **one can talk about probability if and only if the electron is regarded as a particle (= localized entity)** — e.g., the statements:

- “there is equal probability for the electron to pass through each slit”
- “there is probability for the electron to be on a spherical shell around the hydrogen nucleus”

implicitly assume that the electron is a particle.

So, the **experimental evidence** not just demonstrated, but **proved** that probability in quantum mechanics means **existence**, i.e. actual presence of the electron everywhere where its wavefunction is different from zero — e.g.:

equal probability for the electron (implicitly regarded as a particle) to go through both slits means that the entity “electron” does pass through both slits;

equal probability for the electron (implicitly regarded as a particle) to be on a spherical shell around the hydrogen proton means that the entity “electron” does exist as a “cloud” around the proton.

Therefore:

the double-slit experiment **proves** that the entity “electron” passes through both slits and ***no probability is involved***

the lack of a dipole moment of the hydrogen atom in its ground state **proves** that the entity “electron” actually exists around the proton and ***no probability is involved.***

Now the question is:

What an entity the electron is in order to pass through both slits and to exist everywhere on a spherical shell around the hydrogen proton?

These two experimental facts and the Einstein-Podolsky-Rosen argument (in addition to discovering the phenomenon of quantum entanglement) all lead to the realization that ***quantum mechanics is fundamentally incomplete***, not in the sense of EPR, but because it does not tell us what the quantum object ***itself*** is since it deals only with its **states**.

To illustrate the ideas involved let us consider the quantum-mechanical description of the behavior of a particle having a single degree of freedom. The fundamental concept of the theory is the concept of *state*, which is supposed to be completely characterized by the wave function ψ, which is a function of the variables chosen to describe the particle's behavior. Corresponding to each physically observable quantity A there is an operator, which may be designated by the same letter.

EPR, Phys. Rev., 1935

A possible spacetime model of the quantum object will be considered in order to address the apparently paradoxical situations with the double slit experiments, the lack of dipole moment of the hydrogen atom in its ground state and the physical meaning of correlation (not causation) involved in the Einstein-Podolsky-Rosen argument.

This model will also demonstrate that spacetime can equally accommodate deterministic and probabilistic phenomena.

But before discussing the spacetime model of the quantum object we must address the even more fundamental question of the reality of spacetime, because that model would make sense only in a real spacetime.

John Lighton Synge wrote this 65 years ago and, unfortunately, things are pretty much the same now:

“The geometrical way of looking at space-time comes directly from Minkowski. He protested against the use of the word 'relativity' to describe a theory based on an 'absolute' (space-time), and, had he lived to see the general theory of relativity, I believe he would have repeated his protest in even stronger terms... It is to support Minkowski's way of looking at relativity that I find myself pursuing the hard path of the missionary. When, in a relativistic discussion, I try to make things clearer by a space-time diagram, the other participants look at it with polite detachment and, after a pause of embarrassment as if some childish indecency had been exhibited, resume the debate in their own terms.”

J. L. Synge, *Relativity, The general theory* (1960), p. IX

The most famous diagram in the history of Physics!

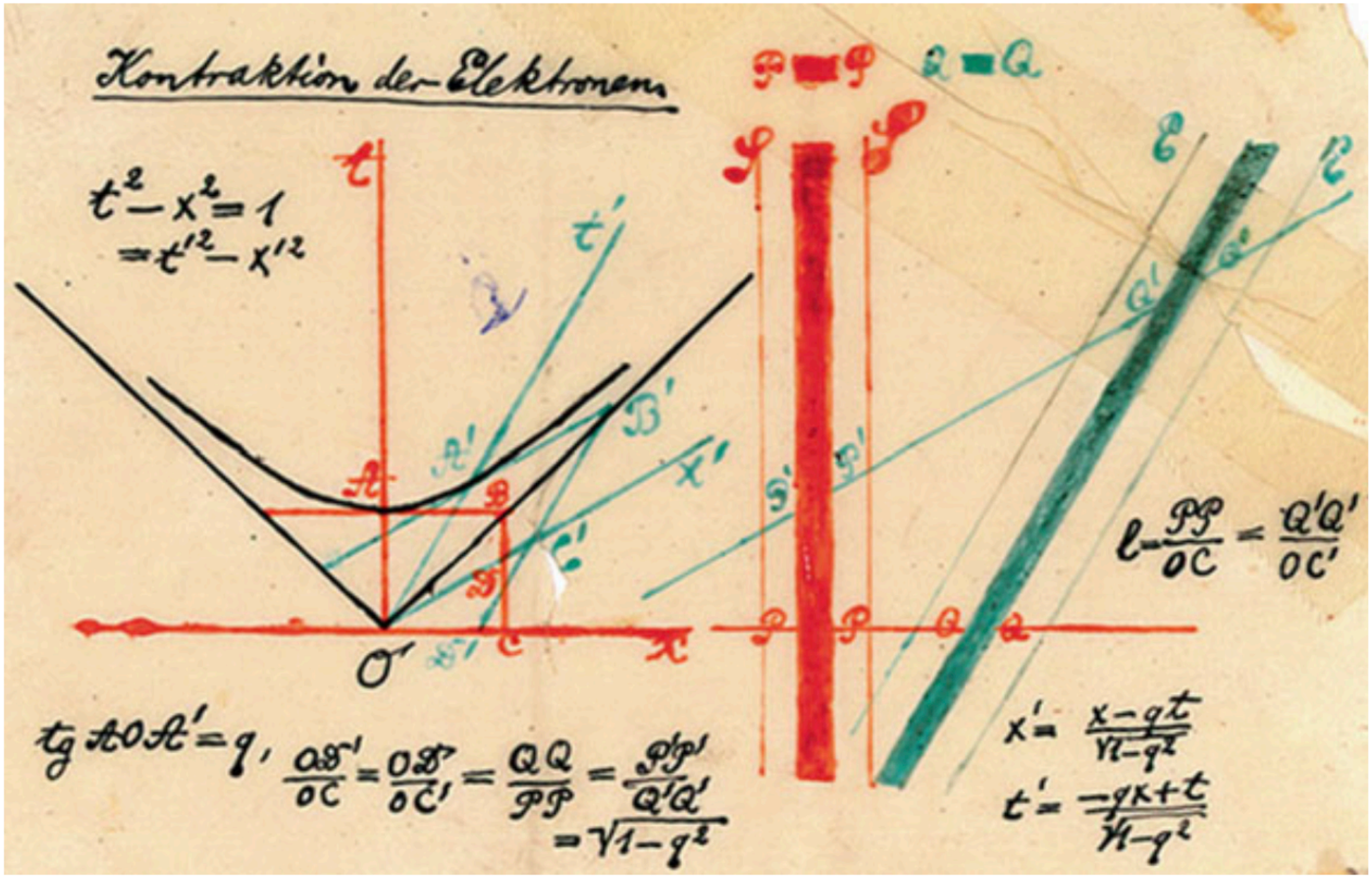


Fig. 1.1 The transparency which Minkowski used at his lecture in Cologne on September 21, 1908. It shows Fig. 1 in his paper "Space and Time." *Source* (with permission): Cover of *The Mathematical Intelligencer*, Volume 31, Number 2 (2009)

V. Petkov, *Seven Fundamental Concepts in Spacetime*, SpringerBriefs in Physics, 2nd ed. (Springer, Heidelberg 2024), Ch. 1

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Now the situation in fundamental physics is truly unprecedented - for over a 100 years most physicists have been ignoring Minkowski's arguments for the reality of spacetime. Their most frequent argument is that the concept of spacetime is just a mathematical space, which does not represent a real four-dimensional world. But, this "explanation" completely and inexplicably ignores the fact that experiments would be impossible if the world were **not** four-dimensional. For example, GPS would not work if the world were **not** four-dimensional.

Answering the question of the dimensionality of the world (i.e., of what the concept of spacetime represents) is of crucial importance for the advancement of fundamental physics because *the physics of a three-dimensional world is radically different from the physics of a four-dimensional world*:

- In a three-dimensional world physics deals with motion and interactions of three-dimensional bodies.
- In a four-dimensional world physics deals with relations between worldlines (or worldtubes in the case of extended bodies) and deformations of curved (non-geodesic) worldtubes because there is no dynamics in spacetime; it is incorrect even to say that a particle follows a timelike geodesic worldline because in spacetime the particle itself is a timelike geodesic worldline.

V. Petkov, *Seven Fundamental Concepts in Spacetime*, SpringerBriefs in Physics, 2nd ed. (Springer, Heidelberg 2024), Ch. 1

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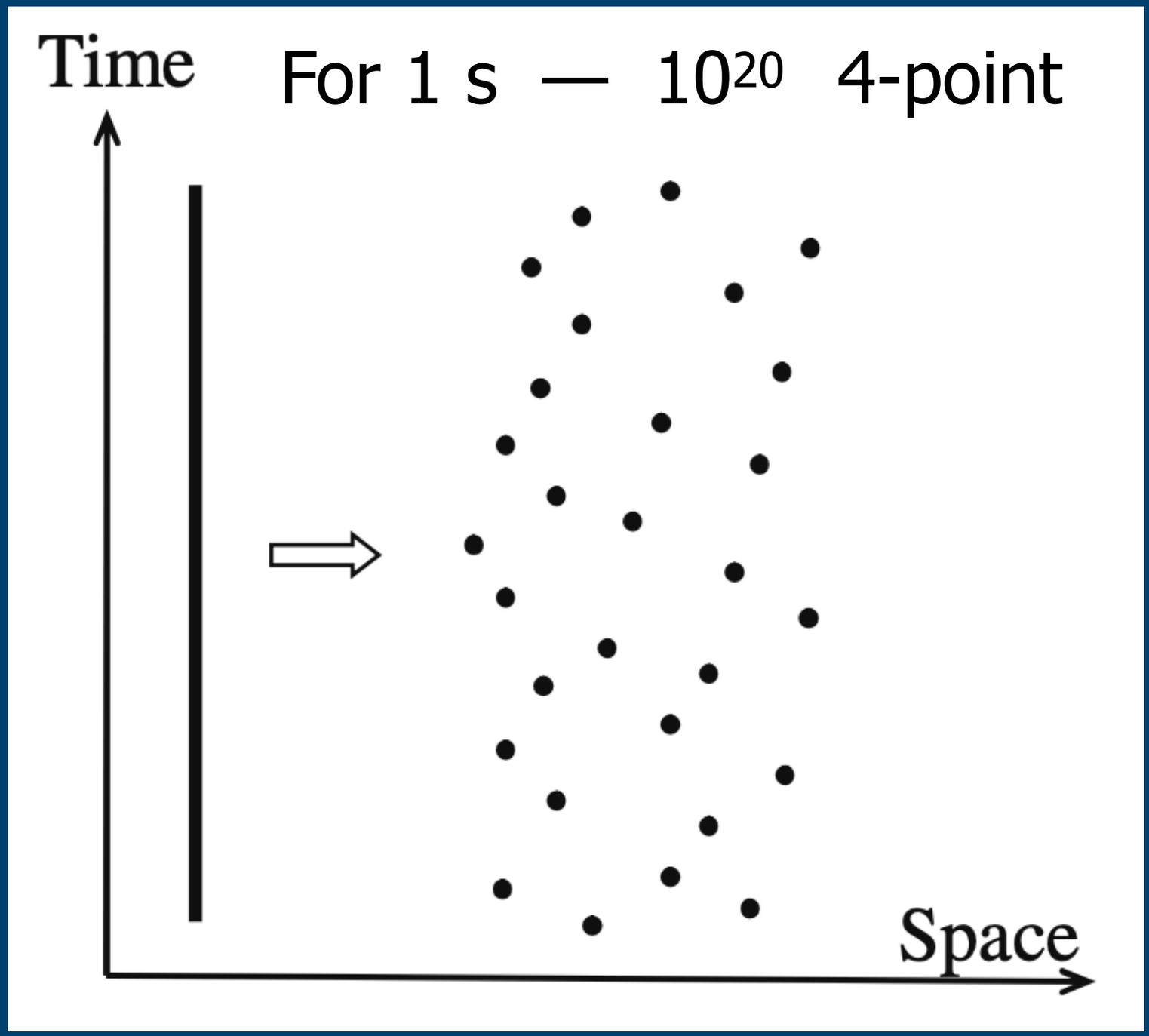
The widespread view that quantum physics and spacetime physics are incompatible is caused by the implicit assumption that electrons (and quantum objects in general) are worldlines in spacetime. But worldlines are not the only thinkable spacetime structures.

If, for the sake of argument, we assume that the quantum object (an electron, for example) is a *different spacetime structure* – atomistic in spacetime (in the spirit of quantum physics). If we assume that the electron's worldline "disintegrates" – then its points (the electron's constituents) will be randomly scattered all over the spacetime region where the electron wavefunction is different from zero.



The idea of 4-atomism. An electron is represented, not by its worldline (the solid line), but by the points constituting the worldline

V. Petkov, *Relativity and the Nature of Spacetime*,
2nd ed. (Springer, Heidelberg 2009), Ch. 10

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In the ordinary three-dimensional language this probabilistic spacetime structure, which brings the idea of atomism to its logical completion (discreteness not only in space, but in time as well — four-atomism or atomism in spacetime), manifests itself as appearance and disappearance of the constituents of the electron at the Compton frequency.

The mysterious Compton frequency of the electron given by the expression $f_C = m_e c^2/h \approx 10^{20} s^{-1}$ can be interpreted as the frequency of appearance / disappearance of the electron's constituents.

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It is unfortunate that this revolutionary idea, put forward by the Bulgarian physicists Anastas Anastassov in the early eighties, remained unnoticed despite that it deals with a fundamental continuity at the heart of quantum mechanics — the almost explicit assumption that quantum objects exist continuously in time.

A. H. Anastassov, *The Theory of Relativity and the Quantum of Action (4-Atomism)*, Doctoral Thesis, University of Sofia, 1984; Self-Contained Phase-Space Formulation of Quantum Mechanics as Statistics of Virtual Particles, *Annuaire de l'Universite de Sofia "St. Kliment Ohridski," Faculte de Physique* **81** (1993), pp. 135–163.

https://vesselinpetkov.com/4-atomism.pdf

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ГОДИШНИК НА СОФИЙСКИЯ УНИВЕРСИТЕТ "СВ. КЛИМЕНТ ОХРИДСКИ"

ФИЗИЧЕСКИ ФАКУЛТЕТ

Том 81, 1993

ANNUAIRE DE L'UNIVERSITE DE SOFIA "ST. KLIMENT OHRIDSKI"

FACULTE DE PHYSIQUE

Tome 81, 1993

SELF-CONTAINED PHASE-SPACE FORMULATION OF QUANTUM MECHANICS AS STATISTICS OF VIRTUAL PARTICLES

ANASTÅS H. ANASTASSOV

Departament of General Physics

https://vesselinpetkov.com/4-atomism.pdf

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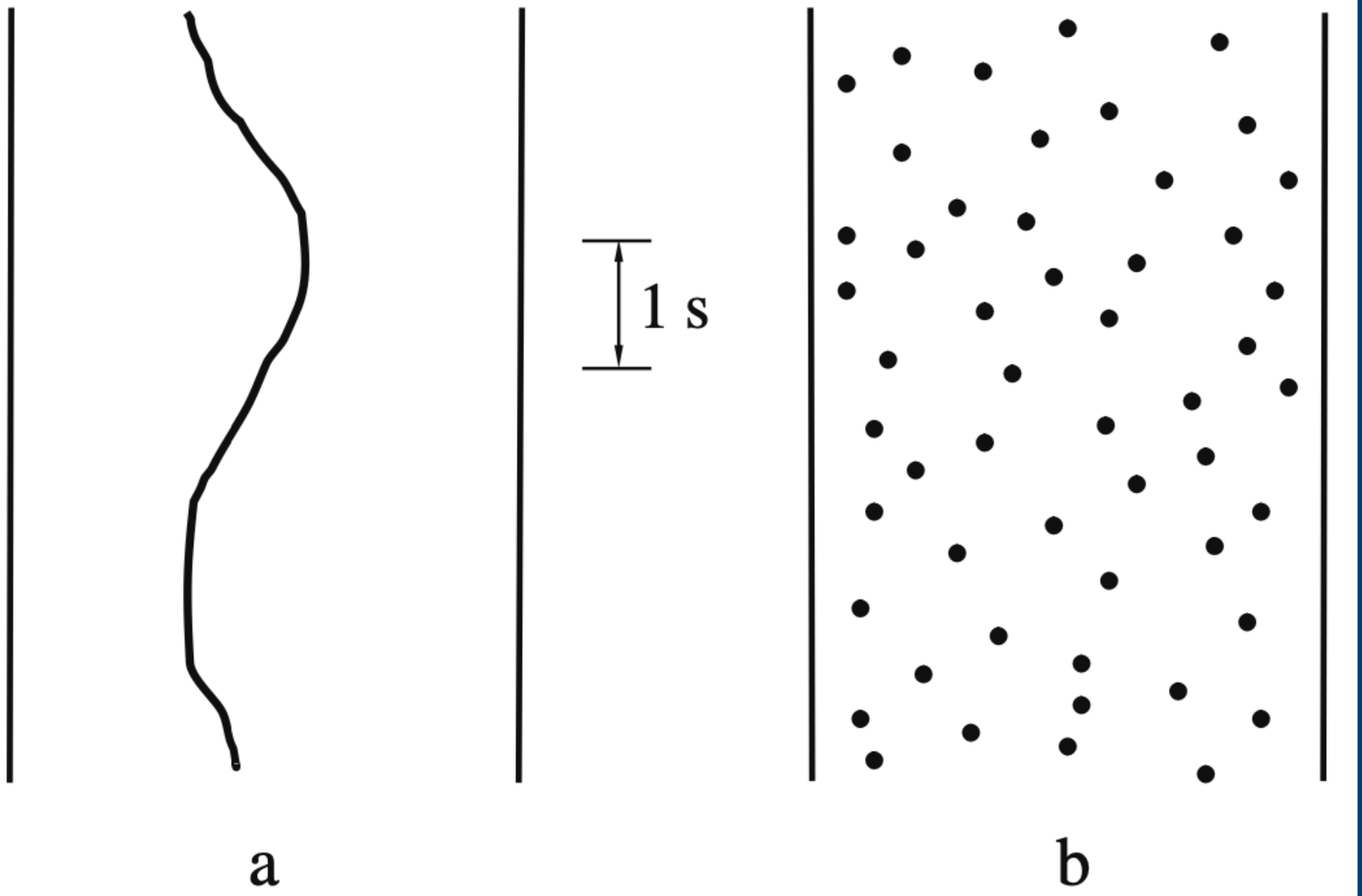
Time



(a) A point-like (classical) electron that exists continuously in time is trapped in a room which is represented by the worldlines of two of its walls. During a given period of time (say, 1 s) such an electron, when not measured, cannot be everywhere in the room, where the probability of finding it is different from zero. (b) During the same period of time an electron which exists discontinuously in time occupies the whole volume of the room.

So, double slit experiments and dipole moment issue are explained.

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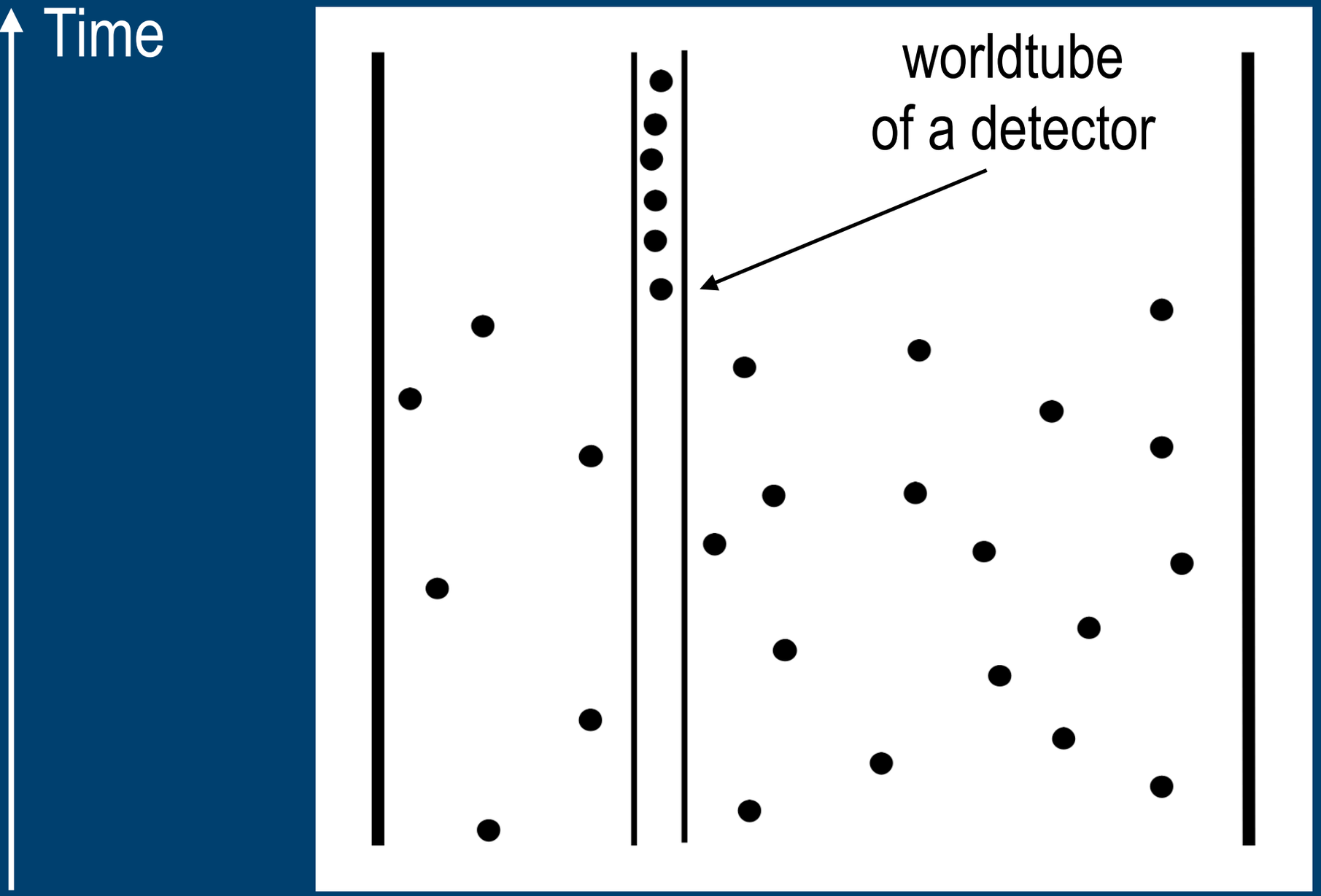


Before being detected, a single electron in a room occupies the whole volume at its disposal. When the first quanton (constituent) appears in the detector, it is trapped there and all subsequent quantons also start to appear and disappear in the detector. This is not an *ad hoc* hypothesis — real particle detectors work in this way (jump in the boundary conditions).

This spacetime model of the electron — four-atomism — (no matter whether or not it will be confirmed) can explain perfectly the probabilistic features of the electron in spacetime: the behaviour of an electron can be described only in terms of probability, but on the other hand the whole history of the electron in time (the probabilistic distribution of all of its “constituents,” representing the electron at all moments of its history) is forever given in spacetime as a “frozen” (forever-given) probabilistic distribution.

Had Minkowski lived longer he might have been thrilled to describe such a probabilistic spacetime structure by the mystical expression “predetermined probabilistic phenomena.”

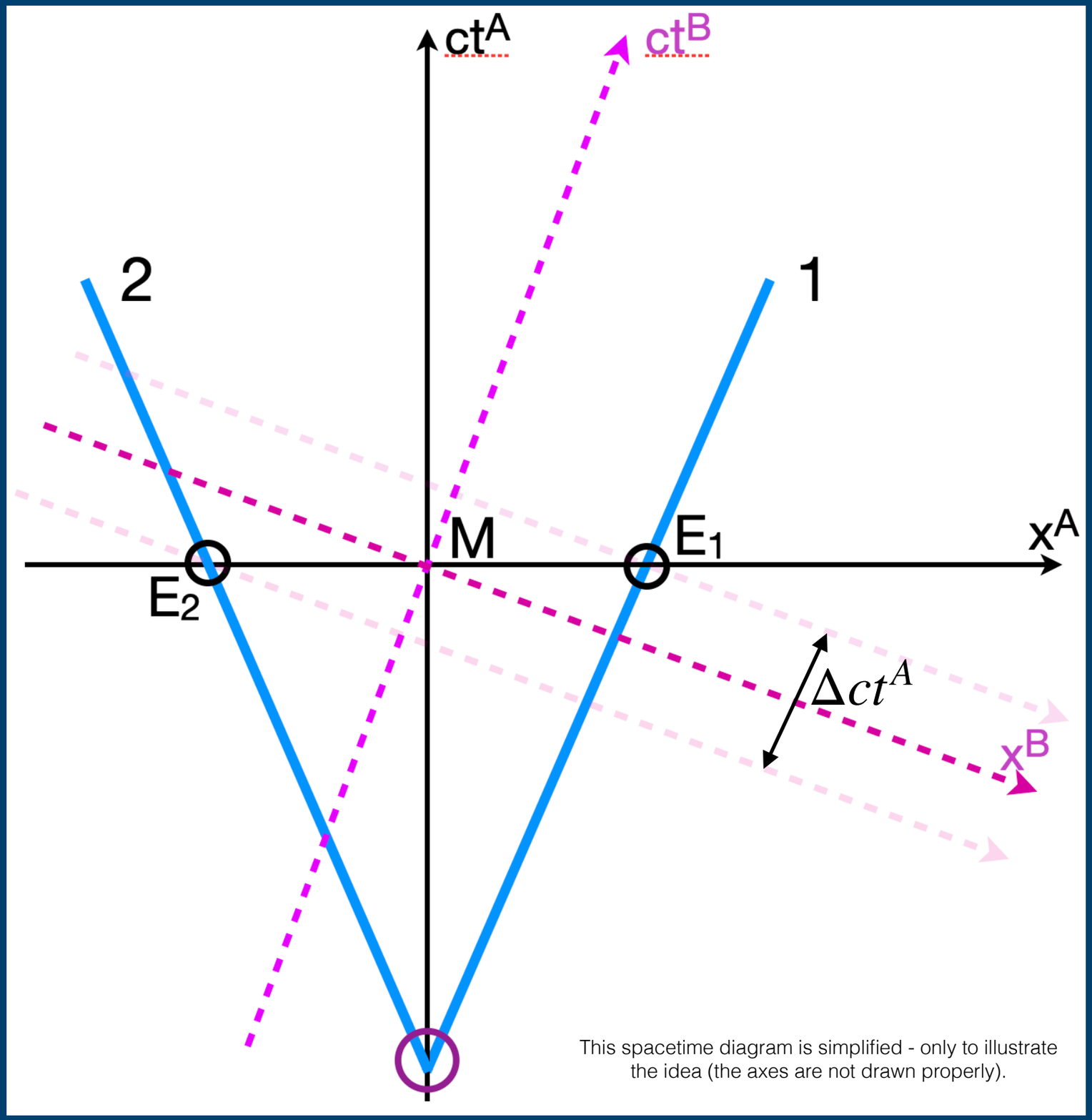
H. Minkowski, “Space and Time,” new translation in: H. Minkowski, *Spacetime: Minkowski's Papers on Spacetime Physics*, edited by V. Petkov (Minkowski Institute Press, Montreal 2020)

From the beginning we can determine the ratio of the units of length and time in such a way that the natural limit of velocity becomes $c = 1$. If we introduce $\sqrt{-1}t = s$ instead of t, then the quadratic differential expression

$$d\tau^2 = -dx^2 - dy^2 - dz^2 - ds^2$$

becomes completely symmetric in x, y, z, s and this symmetry is carried over to any law that does not contradict the world postulate. Thus the essence of this postulate can be expressed mathematically very concisely in the mystical formula:

$$3 \cdot 10^5 \ km = \sqrt{-1} \ seconds.$$



For observer A events E_1 and E_2 are simultaneous, but for observer B event E_2 occurs before event E_1.

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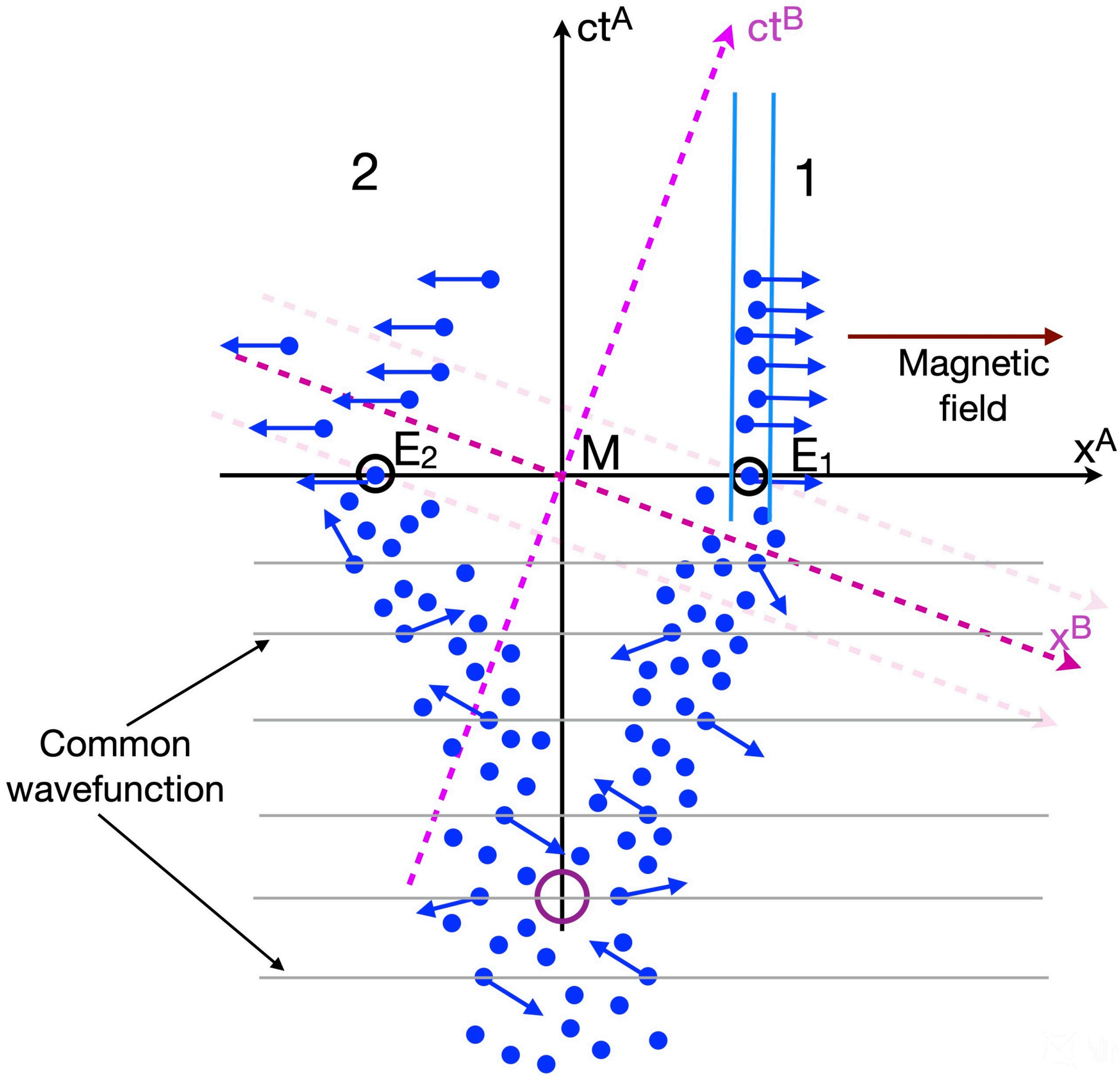

This spacetime diagram is simplified - only to illustrate the idea (the axes are not drawn properly).

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